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INSTRUCTION PAPER
with Examination Questions

FIRST EDITION

Practical Astronomy

PART 2

By I. C. S. Staff

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ADVICE TO THE STUDENT

You learn only by thinking. Therefore, read your lesson slowly enough to think about what you read and try not to think of anything else. You cannot learn about a subject while thinking about other things. Think of the meaning of every word and every group of words. Sometimes you may need to read the text slowly several times in order to understand it and to remember the thought in it. This is what is meant by study.

Begin with the first line on page 1 and study every part of the lesson in its regular order. Do not skip anything. If you come to a part that you cannot understand after careful study, mark it in some way and come back to it after you have studied parts beyond it. If it still seems puzzling, write to us about it on one of our Information Blanks and tell us just what you do not understand.

Pay attention to words or groups of words printed in black-face type. They are important. Be sure that you know what they mean and that you understand what is said about them well enough to explain them to others.

Rules are printed in *italics;* they, too, are important; you should learn to repeat them without looking at the book. With rules are usually given *Examples for Practice.* Work all of these examples according to the rules, but do not send us your work if you are able to get the right answers. If you cannot get the correct answer to an example, send us all of your work on it so that we can find your mistakes. Use one of our Information Blanks.

After you have finished studying part of a lesson, review that part; that is, study it again. Then go on with the next part. When you have finished studying an Instruction Paper, review all of it. Then answer the Examination Questions at the end of the Paper. It is not well to look at these questions until you have finished studying and reviewing the whole Paper.

Answer the Examination Questions in the same order as they are given and number your answers to agree with the question numbers. Do not write the questions. If you cannot answer a question, write us about it on an Information Blank before you send in any of your answers.

Remember that we are interested in your progress and that we will give you by correspondence all the special instruction on your Course that you may need to complete it Remember, too, that you will get more good from your Course if you learn all that you can without asking for help.

[11] INTERNATIONAL CORRESPONDENCE SCHOOLS

PRACTICAL ASTRONOMY

(PART 2)

DETERMINATION OF LATITUDE

GENERAL CONSIDERATIONS

1. Latitude can be most readily determined in the field by measuring the altitude of a heavenly body with either the sextant or the transit. There are three methods, as follows: (1) by observing the altitude of one or more stars when on the meridian; (2) by observing the altitude of the sun when on the meridian; (3) by observing the north star, Polaris, at any time.

When the observer has acquired some experience, he will find that observations on the stars will give more accurate results than can be obtained from the sun. This is because the image of a star in the telescope is a small, well-defined point that permits a more exact setting of the horizontal wire (with the transit), or a more accurate coincidence of the images (with a sextant), than is possible when the sun is observed. Besides, stars suitable for observation can be employed at any hour of the night, while the sun can only be observed for latitude at or near the instant of apparent noon.

By suitably selecting the stars to be observed, the errors of the instrument can be nearly eliminated. This is a very important fact, since the errors of adjustment, especially in the transit instrument, may be so large as to render an accurate determination of the latitude impossible if only a single heavenly body is observed. For these reasons, the

first and the third method should generally be employed in preference to the second.

2. General Formulas for the Determination of Latitude.—From a measured altitude of a heavenly body at the instant of its meridian passage, the latitude of a place is found as follows:

Let Fig. 1 represent a section of the celestial sphere made by the plane of the meridian. The earth is at O; HZH' is the meridian; HH', the horizon; Z, the zenith; EO, the plane of the equator; and P, the north pole. Hence, the arc PH = the arc ZE = the latitude (*Practical Astronomy*, Part 1).

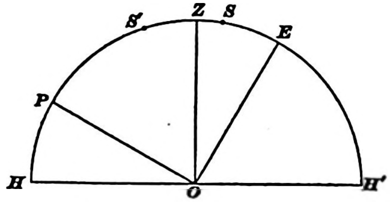

FIG. 1

If the observer is assumed to be in the northern hemisphere, and S is any heavenly body on the meridian, south of the zenith, we have

$$ZE = ZS + SE$$

Or, latitude = declination + zenith distance (for a south star).

If the body is on the meridian north of the zenith, as at S', we have

$$ZE = S'E - S'Z$$

Or, latitude = declination − zenith distance (for a north star).

In applying these formulas, the zenith distance is determined by observing the altitude, correcting it for refraction and index error (and for semi-diameter and parallax, if the sun is observed, and for dip, if the observation is made at sea), and subtracting the result from 90°.

EXAMPLE.—The altitude of the star Sirius was observed at Philadelphia on January 10, 1903; the circle reading was 66° 56′ 50″, the index error was +22″. To find the latitude of Philadelphia.

SOLUTION.—

Observed double altitude	66° 56′ 50″
Correction for index error	+22
Corrected double altitude	66° 57′ 12″

Apparent altitude	33° 28′ 36″
Refraction (Table VII, *Practical Astronomy*, Part 1)	−1 26
True altitude	33° 27′ 10″
Zenith distance = 90° − true altitude	56° 32′ 50″
Declination of Sirius from Ephemeris (Table V, *Practical Astronomy*, Part 1)	−16 34 58
Latitude	39° 57′ 52″ Ans.

FIRST METHOD OF DETERMINING LATITUDE: BY OBSERVING A STAR ON THE MERIDIAN

3. Selection of Stars.—Before going into the field, the observer should select at least two stars for observation, one of which passes north and the other south of the zenith. The altitudes of the two stars when on the meridian should be as nearly equal as possible. If three or four such pairs of stars are observed, and the mean of the resulting six or eight values of the latitude is formed, the value obtained will be much more accurate than could have been derived from a single pair. The disagreement of the individual values of the latitude will serve to indicate to the observer what the accuracy of his observations is.

The observer, after deciding at what time in the evening he will begin his observations, should first change this mean time into sidereal time. Since the right ascension of a star is equal to the sidereal time of its transit (*Practical Astronomy*, Part 1), he should then find, from the Ephemeris (Table V, *Practical Astronomy*, Part 1), a star whose right ascension is, as nearly as possible, equal to the sidereal time chosen. If in the northern hemisphere, the declination of the star is greater than the latitude [which usually is known within a few degrees, or can be approximately found, by observing the altitude of Polaris (Art. 2)], the star will pass north of the zenith; but if the declination is less, the star will pass south of the zenith (Art. 2). After selecting one star, another having approximately the same altitude when on the meridian may be found as follows:

Let l' be the approximate latitude of the place, d_n and z_n the declination and zenith distance, respectively, of a north star; d_s and z_s similar quantities for a south star. Then (Art. 2),

$$d_n = z_n + l',\ d_s = l' - z_s$$

If a north star is chosen as the first star, the second, or south, star must be so chosen that z_s will be approximately equal to z_n, and that, therefore, its declination will be approximately equal to $l' - z_n$. Similarly, if a south star is chosen as the first star, the second, or north, star must be so chosen that its zenith distance z_n will be approximately equal to z_s and that, therefore, its declination will be approximately equal to $z_s + l'$.

The stars selected should be bright (say of the first, second, or third magnitude). After selecting the first, the second may be selected by looking down the list of stars (Ephemeris, or Table V, *Practical Astronomy*, Part 1) until a bright star is found having approximately the required declination, as determined above.

EXAMPLE.—To select a pair of stars for observation at Philadelphia on January 5, 1903, the longitude being $-7^m\ 37^s$ from Washington and $5^h\ 0^m\ 38.5^s$ from Greenwich, and the latitude about 40°. The observer possesses an ordinary watch that keeps 75th-meridian time (that is, the local time of places whose longitude from Greenwich is 75°, or 5^h), and is 20 seconds fast.

SOLUTION.—Since Philadelphia is $5^h\ 0^m\ 38.5^s$ west of Greenwich, a watch keeping standard 75th-meridian time is 38.5^s fast on local time. Hence, the watch used is actually $38.5^s + 20.0^s = 58.5^s$ fast on local mean time.

Suppose that it is desired to begin observations about 6 P. M. From the complete table of the Ephemeris, of which a part is given as Table IV in *Practical Astronomy*, Part 1, we find, sidereal time of mean noon at Washington = $18^h\ 56^m$. Sidereal time corresponding to 6^h after mean noon is, roughly, $24^h\ 56^m$, that is, $0^h\ 56^m$. Looking now at the list of stars in the Ephemeris, or Table V, *Practical Astronomy*, Part 1, it is found that the star γ Cassiopeia is well situated for observation at about this time; its magnitude is but 2.3, so that it is quite bright, and as its declination is +60°, it is not too near the horizon (*Practical Astronomy*, Part 1). The latitude of Philadelphia is about 40°; hence, γ Cassiopeia is a north star (Art. 3). To find the approximate zenith distance when on the meridian, the latitude 40° is subtracted from the

approximate declination (say 60° 11′), and the zenith distance of the first star is found to be 20° 11′. The zenith distance of the second star must also be about 20°, and hence its declination must be about $40° - 20° = +20°$. Looking on down the list, the stars β Arietis and α Arietis, both of which fulfil this condition, are found. The latter is the brighter of the two, and is therefore selected.

A small table to be taken into the field is then prepared as follows:

Name of Star	Right Ascension	Declination	Magnitude	Watch Time of Transit	Altitude of Transit
γ Cassiopeia	$0^h\ 50^m\ 51^s$	+60° 11′ 30″	2.3	$5^h\ 54^m\ 24.5^s$	69° 49′ N
α Arietis . .	2 1 42	+23 0 14	2.1	7 5 3.5	72 0 S

The first four columns are copied from the Ephemeris. To find the exact watch time of transit of γ Cassiopeia, the sidereal time $0^h\ 50^m\ 51^s$ is changed into local mean time (*Practical Astronomy*, Part 1), and to the result is added 58.5^s, the watch error. (This computation requires the complete tables of the Ephemeris; Table IV, *Practical Astronomy*, Part 1, is not sufficiently extended for the purpose.)

The last column is found by subtracting the latitude from the declination for the north star to obtain the zenith distance, and by subtracting the declination from the latitude for the south star, and finally subtracting the two zenith distances from 90° to obtain the altitudes.

METHODS OF OBSERVING

4. Observation With a Sextant.—The approximate value of the double altitude of the first star at the time of transit is set off on the limb of the instrument, and the telescope is directed to the image of the star in the mercury. The two images that will be seen in the field of the telescope are kept in coincidence by turning the tangent screw until the instant of meridian passage is shown by the watch. The reading is then taken; this is the desired (uncorrected) double altitude.

If the time is not known with accuracy, the method of procedure is as follows: The images are brought into coincidence, as before, a few minutes before the star reaches the meridian, and kept in coincidence by turning the tangent screw. Since the altitude of the star when on the meridian is its greatest altitude, the turning of the tangent screw is

stopped the instant that the readings cease to increase and begin to decrease. The resulting reading is, as before, the uncorrected double altitude of the star when on the meridian.

5. **Observation With a Transit.**—The adjustments of the transit having been carefully attended to, there are three ways of proceeding, according as the vertical circle is or is not complete, and as a mercury horizon is or is not used:

1. *If the vertical circle is not complete.*—The index error must be well determined according to directions given in *Hydrographic Surveying.* Then the horizontal wire is brought on the first star, and the vertical motion clamped. The horizontal wire is kept constantly bisecting the image of the star by turning the tangent screw until the star ceases to rise; the instant when the upward motion of the star ceases and the star begins to fall, the turning of the tangent screw is stopped. The reading of the vertical circle, corrected for index error and for refraction, is the desired meridian altitude.

2. *If the vertical circle is complete.*—The observation is begun 3 or 4 minutes before the time of meridian passage, and the altitude of the star is measured by the method given in *Practical Astronomy*, Part 1. The result should be corrected for refraction, but not for index error.

3. *If an artificial horizon is used.*—The method given in *Practical Astronomy*, Part 1, is employed. The readings are usually made in the following manner: The transit is directed to the star and the vertical circle read; two sightings are taken on the mercury image, and the corresponding angles read from the vertical circle; finally, one more reading on the star is taken. By this time the star should be about 5 minutes past the meridian. The mean of the direct plus the mean of the reflected readings will be the double altitude, which should be divided by two and corrected for refraction, but not for index error.

The artificial horizon must be moved to a new position before observing the second star, and therefore the two stars should be so selected that there will be an interval of

at least 10 or 15 minutes between the times at which they come to the meridian. In other words, their right ascensions should not differ by less than this amount.

EXAMPLES FOR PRACTICE

1. The altitude of Polaris was measured on January 2, 1903, with a transit having an incomplete vertical circle; the circle reading was 50° 12′ 0″; the correction due to index error was +2′. Find the latitude.
Ans. +49° 0′ 54″

2. The altitude of Polaris was measured on January 5, 1903, as described in *Practical Astronomy*, Part 1; the vertical circle readings were as follows: Telescope sighted at star, 36° 27′ 10″; telescope sighted at reflection in mercury, 323° 32′ 30″; telescope sighted at reflection in mercury, 323° 33′ 0″; telescope sighted at star, 36° 27′ 10″. Find the latitude.
Ans. +35° 13′ 38″

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6. Accuracy of the Preceding Methods of Observing.—In general, the latitude can be determined from a single altitude about as accurately as the circles can be read. The principal sources of error are refraction, index error, and want of adjustment of the instrument. If the star is not too low, the uncertainty of the refraction seldom amounts to more than 1 or 2 seconds, and if the second or third method is employed, the other errors are almost wholly eliminated.

If a very exact determination is required, three or four pairs of stars at varying altitudes should be observed, and the mean of the six or eight resulting values of the latitude should be taken. This result should be accurate within 5″ with the sextant and within 5″ to 20″ with the transit. The error to be expected depends largely on the excellence of the instrument employed.

MODIFICATION OF THE FIRST METHOD FOR VERY ACCURATE WORK

7. The modification consists in measuring several altitudes of a star when it is near the meridian, and applying a correction for the change of altitude that takes place during the observations. This correction is obtained from Table I

at the end of this Section. If this method is employed, the declination of the star must be less than $+24°$ or greater than $-24°$.

8. Directions for Observing.—Not more than 12 minutes before the star reaches the meridian, the observer should begin to measure repeatedly its altitude by the methods of Arts. 4 or 5, and should record the exact time when each observation is taken. He should measure ten, twenty, or thirty altitudes in this manner, taking care not to prolong the observations for more than 12 minutes after the time of meridian passage.

9. Reduction of the Observations.—The altitude of the star is continually increasing until the time of meridian passage; after this it begins to decrease. If the observations are pursued without interruption, the time at which the greatest altitude is recorded is the time of meridian passage. If the observations are interrupted at this time, or if all the altitudes are measured before or after the star passes the meridian (as is sometimes done), the watch time of meridian passage must be computed (Art. 3). The difference between each recorded time and the time of meridian passage is then determined, and the squares of the *time intervals* so obtained are to be taken from Table II at the end of this Section.

10. Change in Altitude.—The amount by which the altitude of the star changes in 1 minute is then determined. This quantity is found in Table I, and its value corresponding to the given declination and the approximate latitude may be taken out directly. The change in 1 minute is then multiplied by each square interval, and the products are added to the corresponding measured altitudes; the resulting sums will be the corrected meridian altitudes. The mean of these sums is next formed and corrected for index error and refraction. The result is the meridian altitude as obtained from the ten, twenty, or thirty measurements. The latitude is then found in the usual manner.

EXAMPLE.—The following observations were made on α Aquilæ by Lewis Boss in connection with the northern boundary survey of the United States:

Number	Circle Reading	½ Circle Reading	Watch Time
1	99° 5′ 35″	49° 32′ 47.5″	20^h 1^m 35^s
2	6 10	33 5	2 37
3	7 5	33 32.5	3 57
4	7 55	33 57.5	5 5
5	8 10	34 5	6 41
6	8 0	34 0	7 52
7	7 50	33 55	8 51
8	7 40	33 50	9 47
9	7 5	33 32.5	10 41
10	99 6 55	49 33 27.5	20 12 0

A sextant was used of which the index error was −3′ 43″. The approximate latitude was +49°, and the declination of α Aquilæ, +8° 32′ 11.5″. To find the true latitude.

SOLUTION.—The time at which the star had its maximum altitude was 20^h 6^m 41^s, which is therefore the time of meridian passage. The differences between this time and the recorded times are first formed and placed in the second column of the following table. The squares of the intervals found in column three are taken directly from Table II at the end of this Section.

Number	Interval From Time of Meridian Passage	Square of Interval	Product	Meridian Altitudes
1	5^m 6^s	26.0	52.0	49° 33′ 39.5″
2	4 4	16.5	33.0	33 38.0
3	2 44	7.5	15.0	33 41.5
4	1 36	2.6	5.2	34 2.7
5	0 0	0.0	0.0	34 5.0
6	1 11	1.4	2.8	34 2.8
7	2 10	4.7	9.4	34 4.4
8	3 6	9.6	19.2	34 9.2
9	4 0	16.0	32.0	34 4.5
10	5 19	28.3	56.6	49 34 24.1

The numbers in the third column are then multiplied by the number corresponding to latitude 49° and declination +8.5° from Table I. This number is found to be 2.0; the products are put in the

fourth column. Finally, the products of column four are added to the corresponding single altitudes; the sums are found in column five, and these are the final meridian altitudes. The mean of these ten values is determined next, this being the meridian altitude as determined from all the measurements.

Final observed meridian altitude	49° 33′ 59.8″
Index correction = ½ × (−3′ 43″)	−1 51.5
Refraction (Table VII, *Practical Astronomy*, Part 1)	−48.5
True altitude	49° 31′ 19.8″
Zenith distance = 90° − true altitude	40° 28′ 40.2″
Declination of α Aquilæ	+8 32 11.5
Latitude	49° 0′ 51.7″ Ans.

SECOND METHOD OF DETERMINING LATITUDE: BY OBSERVING THE SUN ON THE MERIDIAN

11. Since the sun is in motion among the stars, its declination is constantly either increasing or decreasing, and hence the sun does not attain its maximum altitude exactly at noon. From this cause, a slight error enters into the method now to be explained, and when this method is used, the errors of the sextant or transit cannot be completely eliminated, as in the methods explained in Arts. **3** and **4**. Nevertheless, when the latitude is not required with an accuracy greater than 20″ to 30″, observations on the sun are often the most convenient for determining the latitude. Two cases will be described, namely: (1) observation of a single altitude when the sun is on the meridian; and (2) observations of two or more altitudes when the sun is near the meridian.

12. Case I.—Some 10 or 15 minutes before noon, the observer should begin to observe the lower edge of the sun with either the sextant or the transit. He should repeat this observation every minute or two. At first, the altitude will be increasing, but immediately after noon it will begin to decrease. The maximum altitude obtained is the meridian altitude, and when this is corrected for index error, refraction, parallax, and semi-diameter, the true altitude of the sun's center will be obtained.

When the local mean time is known within a minute or two, the approximate time of the sun's meridian passage may be obtained by subtracting from the mean time the equation of time taken from the Ephemeris (*Practical Astronomy*, Part 1). The observation may then be begun 1 or 2 minutes before apparent noon.

13. To Find the Watch Time of Apparent Noon When the Watch Is Running on Standard Time and Has a Known Error.—The local apparent time of noon is $12^h\ 0^m\ 0^s$. Adding to this the equation of time, the local mean time of the sun's meridian passage is obtained. To this sum must be added the difference in longitude between the place of observation and the standard meridian, if the place of observation is west of the standard meridian; otherwise, that difference should be subtracted. The result is the standard time of apparent noon. The known error of the watch is then added or subtracted, as the case may be, and the result is the desired watch time.

EXAMPLE.—To find the time of apparent noon at Philadelphia, January 10, 1903, as shown by a watch that is 52^s fast on standard 75th-meridian time, Philadelphia being $5^h\ 0^m\ 39^s$ west of Greenwich.

SOLUTION.—

Apparent solar time	$12^h\ 0^m\ 0^s$
Equation of time from Ephemeris, or from Table IV, *Practical Astronomy*, Part 1	$+7\ \ 25$
Local mean time of apparent noon	$12^h\ 7^m\ 25^s$
Philadelphia is west of principal meridian	$+39$
Standard time of apparent noon	$12^h\ 8^m\ 4^s$
Error of watch	$+52$
Watch time of apparent noon	$12^h\ 8^m\ 56^s$ Ans.

EXAMPLES FOR PRACTICE

1. Find the standard time of apparent noon at Washington on January 10, 1903, the longitude of Washington being $5^h\ 8^m\ 16^s$ from Greenwich. Ans. $12^h\ 15^m\ 41^s$

2. Find the watch time of apparent noon at Philadelphia on January 10, 1903, the watch being $3^m\ 10^s$ slow on standard time, and the longitude of Philadelphia being $+5^h\ 0^m\ 39^s$. Ans. $12^h\ 4^m\ 54^s$

14. Case II.—Either the sextant or the transit may be used, and the watch time of each observation must be recorded. If the sextant is employed, the observations should begin, if possible, 5 or 10 minutes before apparent noon, and be continued as long as may be necessary to make the sun's meridian passage occur about the middle of the series. If the transit is employed, very good results may be obtained by the methods of observing explained in *Practical Astronomy*, Part 1. Two or four such series should be taken, but in general not more. In any case, one-half the altitudes should be measured on the upper edge of the sun and one-half on the lower, in order to avoid the correction for semi-diameter. The measurements are reduced by the method of Art. 9.

The following example will fully illustrate the method of recording the observations and of computing the latitude:

EXAMPLE.—The observations tabulated below were made by the method given in *Practical Astronomy*, Part 1. The instruments were an artificial horizon, a transit instrument having a complete vertical circle, and a mean-time watch. The watch time of transit was $11^h\ 54^m\ 17.5^s$ (Art. 14); the approximate latitude was $+48°\ 2'$; and the declination of the sun (*Practical Astronomy*, Part 1) was $-14°\ 7'\ 18.1''$. To find the true latitude.

RECORD OF OBSERVATIONS

Edge of Sun	Pointing	Reading of Vertical Circle	Watch Time
Upper	Direct	28° 14′ 20″	$11^h\ 50^m\ 44^s$
	Mercury	331 46 0	51 42
	Mercury	331 46 20	52 28
	Direct	28 15 20	53 50
Lower	Direct	207 41 50	56 32
	Mercury	152 18 40	58 24
	Mercury	152 18 40	59 56
	Direct	207 41 30	12 1 14

NOTE.—The readings of the circle are here given exactly as they were recorded. It will be noticed that the graduation with this transit was from 0° to 360°. The instrument was turned 180° in azimuth and the telescope inverted between the two series of measurements, in order to completely eliminate all errors of adjustment.

SOLUTION.—	FIRST SERIES	SECOND SERIES
Mean of readings on the sun direct	28° 14′ 50″	207° 41′ 40″
Mean of readings on sun reflected from mercury	331 46 10	152 18 40
Double altitude of edge of sun . .	56° 28′ 40″	55° 23′ 0″
Single altitude of edge of sun . . .	28° 14′ 20″	27° 41′ 30″

	Measured Altitudes	Time	Interval	Square	Product	Meridian Altitude
Upper edge	28° 14′ 20″	$11^h 52^m 11.0^s$	$2^m 6.5^s$	4.4	6.2″	28° 14′ 26.2″
Lower edge	27 41 30	11 59 1.5	4 44.0	22.4	31.4	27 42 1.4

The column headed Time contains the mean of the four times recorded while observing the upper and lower edges, respectively. The differences between these two means and the watch time of apparent noon ($11^h 54^m 17.5^s$) is the Interval; the square of this interval is taken from Table II. From Table I is next taken the number corresponding to a latitude of 48° 2′ and a declination of 14° 7′ of a "different name" (that is, different sign) from the latitude. This is found to be 1.4, and it is multiplied by the squares to give the desired corrections, which are written in the column headed Product. These products are finally added to the Measured Altitudes of the second column to form the Meridian Altitudes. The solution is then completed as follows: Mean of the meridian altitudes is equal to the altitude of the sun's center, which need not be corrected for semi-diameter, nor for index error (*Practical Astronomy*, Part 1) 27° 58′ 13.8″

Refraction (Table VII, *Practical Astronomy*, Part 1)	−1 47.0
Parallax (Table VIII, *Practical Astronomy*, Part 1)	+8.0
Corrected altitude of sun's center	27° 56′ 34.8″
Zenith distance = 90° − altitude	62° 3′ 25.2″
Declination of sun	−14 7 18.1
Latitude	47° 56′ 7.1″ Ans.

THIRD METHOD OF DETERMINING LATITUDE: BY OBSERVING POLARIS

15. As the north star is near the pole of the heavens and is easily identified in the sky, even by an inexperienced observer, it is frequently recommended in textbooks on astronomy that this star be used for the determination of latitude. The observations may be made by one of two methods: (1) star on the meridian; (2) star in any position.

16. The star may be observed on the meridian exactly as described in Arts. **1** to **6**. For determining the time of meridian passage, the right ascension, which may be taken from the Ephemeris, is necessary, and for finding the resulting latitude, the declination must be known. A simple method of finding the time of the meridian passage of Polaris without recourse to its right ascension is given in *Transit Surveying*, Part 2. The declination must, however, be taken from the Ephemeris.

17. If the observer possesses an Ephemeris, he may determine his latitude by observing the altitude of Polaris at any time, and using the tables of that work entitled "For Finding the Latitude by Polaris." The simple directions for using them are given in full. Even the tables of the Ephemeris are, however, liable to an error of 30″ or more in the resulting latitude. If a higher degree of accuracy is required, the following rule should be applied:

Rule.—*Convert the mean time of the observation into sidereal time, and from the result, subtract the right ascension of Polaris; the remainder is the hour angle of the star. Reduce the hour angle to arc measure.*

Subtract the declination from 90°; the result is the polar distance.

Correct the measured altitude for refraction and for index error, if the latter has not been eliminated in making the observations. From the corrected altitude, subtract the product of the polar distance by the cosine of the hour angle, and to the

remainder, add the second correction taken from Table III at the end of this Section; the result is the desired latitude.

EXAMPLE.—The altitude of Polaris was measured ten times with a transit having an incomplete vertical circle, by the method of Art. **16**. The mean of the circle readings was 39° 33′ 50″; the mean of the corresponding recorded times, reduced to sidereal time, was $10^h\ 45^m\ 8.9^s$. The index error was +57.4″. The right ascension and declination of Polaris, taken from the Ephemeris, were $1^h\ 15^m\ 6.0^s$ and 88° 41′ 6.2″, respectively. To find the latitude.

SOLUTION.—

Measured single altitude	+39° 33′ 50.0″
Correction for refraction (Table VII, *Practical Astronomy*, Part 1)	−1 8.6
Index error	+57.4
Corrected true altitude	39° 33′ 38.8″

Polar distance = 90° − declination = 1° 18′ 53.8″ = 4,733.8″.

Hour angle = sidereal time − right ascension = $9^h\ 30^m\ 2.9^s$ = 142° 30′ 43.5″.

Polar distance × cosine of hour angle = 4,733.8 × −.79348	−1° 2′ 36.2″
True altitude	39 33 38.8
Altitude-product	40° 36′ 15.0″

The second correction, from Table III, corresponding to $10^h\ 45^m$ and 39° 30′, is 15.2″. Adding this correction to the altitude-product, the resulting latitude is found to be 40° 36′ 30.2″. Ans.

DETERMINATION OF TIME

18. The problem of the determination of time in astronomy always consists in determining the *error* of a chronometer or clock. The error of a clock is the number of hours, minutes, and seconds that must be subtracted from the time shown by the clock to obtain the true time. If the clock indicates a time later than the true time, it is said to be fast; if it indicates a time earlier than the true time, it is said to be slow. When a clock is fast, its error is considered positive; and when it is slow, its error is negative. Thus, if at the instant of mean local noon a watch indicates $12^h\ 5^m\ 10^s$, the watch is fast $5^m\ 10^s$, and its error on local mean time is $+5^m\ 10^s$. To obtain the true time from that of such watch at any instant, $+5^m\ 10^s$ must be subtracted.

In general, if

T = true time at any instant;

T_c = time shown by clock at same instant;

e = clock error;

we have, $e = T_c - T$, $T = T_c - e$, $T_c = T + e$

These equations apply whether the clock is keeping local, standard, or sidereal time.

EXAMPLE 1.—The error of a watch on local mean time is $+2^m\ 4^s$. What is the true time when the watch reads 9 A. M.?

SOLUTION.—

$$\begin{array}{rl} T_c = & 9^h\ 0^m\ 0^s \text{ A. M.} \\ e = & \underline{\quad +2\ \ \ 4} \\ T = & 8^h\ 57^m\ 56^s \text{ A. M. Ans.} \end{array}$$

EXAMPLE 2.—The error of a watch is -40^s. What is the watch time of true local mean noon?

SOLUTION.—

$$\begin{array}{rl} T = & 12^h\ 0^m\ 0^s \\ e = & \underline{\qquad\quad -40} \\ T_c = & 11^h\ 59^m\ 20^s \text{ Ans.} \end{array}$$

EXAMPLES FOR PRACTICE

1. A clock is 20 seconds fast. What is the true time when the clock indicates 6^h? Ans. $5^h\ 59^m\ 40^s$

2. The right ascension of a star is $11^h\ 2^m\ 8.04^s$; the error of a sidereal clock is -11.08^s. Find the clock time of the star's transit. Ans. $11^h\ 1^m\ 56.96^s$

3. A watch keeping mean time has an error of $-1^m\ 10^s$; the equation of time is $-16^m\ 4.01^s$. Find the watch time when the sun is on the meridian. Ans. $11^h\ 42^m\ 45.99^s$

4. A sidereal clock has an error of $+16.02^s$; when a star was on the meridian the clock time was $20^h\ 8^m\ 19.22^s$. What was the right ascension of the star? Ans. $20^h\ 8^m\ 3.20^s$

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19. In *Practical Astronomy*, Part 1, it was shown that the apparent solar time at any instant is simply the hour angle of the center of the sun at that instant, and that the sidereal time may be found by adding the observed hour angle of any star to its right ascension. Hence, the mean time at any instant may be found in one of two ways: (1) by determining

the hour angle of the sun's center and adding to this the equation of time; (2) by determining the hour angle of any star, adding to it the star's right ascension, and finally changing the resulting sidereal time into mean time. The hour angle of any heavenly body is found by measuring the altitude of the body and applying a trigonometric formula, as will be explained farther on.

FIRST METHOD OF DETERMINING TIME: FROM OBSERVATIONS ON THE SUN

20. Method of Making the Observations.—The observations should never be made later than 10 A. M. nor earlier than 2 P. M., and in general the less the altitude of the sun the better; though the altitude should not be less than 15°, since the correction for refraction then becomes uncertain.

If the observer uses a sextant, he first brings the edges of the direct and reflected images into contact. He next moves the index of the vernier forwards on the limb if the sun is rising, backwards on the limb if the sun is setting, 10′, 20′, or 30′, until it stands at an even reading. He then directs the telescope again to the mercury, when he will see two images either slightly separated or slightly overlapping, according as he is observing the upper or the lower edge of the sun. In the first case, the images will appear to be coming together, as at (*b*), Fig. 2; in the second case, to be slowly separating, as at (*a*). The instant they are seen to become tangent to each other, as at (*c*), the observer notes the exact time and the circle setting. The vernier is then moved forwards or backwards, as the case may be, 10′, 20′, or 30′, and a second observation is taken. Five such measurements of altitude usually constitute a series. If two series are taken, one on the upper and one on the lower edge of the sun, no correction for semi-diameter is necessary.

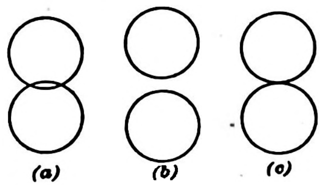

FIG. 2

21. If a transit is used, the telescope is first directed to the sun, and the horizontal wire placed tangent to the upper or to the lower edge. The index of the vernier is then moved forwards on the vertical circle 20′ or 30′, if the sun is rising, backwards if the sun is setting, until it stands at an even reading.

If the lower edge is observed, the appearance in the telescope will be either that of Fig. 3 (*a*), if the sun is rising, or that of (*c*), if it is setting; while if the upper edge is observed, the appearance will be as at (*b*) in the afternoon, and as at (*d*) in the forenoon. In (*c*) and (*d*), the sun is approaching the cross-wire; in (*a*) and (*b*), it is moving off the cross-wire. The instant the cross-wire is seen to be tangent to the disk, as in (*e*) or (*f*), the observer notes and records

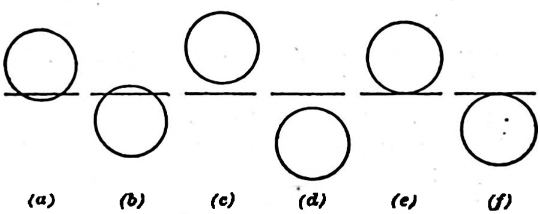

FIG. 3

the time, and also the reading of the vernier on the circle. The vernier is then moved forwards or backwards 10′, 20′, or 30′, and the observation is repeated. Five such observations usually constitute a series.

If one series is taken with the telescope direct and a second series with the telescope reversed, the index error and the errors of adjustment will be eliminated. If, in addition, one-half of the observations are made on the upper edge of the sun and the other half on the lower edge, no correction for semi-diameter need be applied. Finally, the mercury horizon may be used, and each altitude measured, as explained in *Practical Astronomy*, Part 1.

22. Method of Observing the Time.—The observer should place the clock or chronometer in such a position

that its beats can be heard distinctly. The instant the images are in contact, Fig. 2 (*c*), or the disk tangent to the wire, Fig. 3 (*e*) and (*f*), he should begin to count the clock beats and continue to count them until he can take the clock reading. The interval of time corresponding to the number of ticks counted until the time was read from the clock is to be subtracted from the clock reading to give the instant of observation.

23. To Find the True Altitude.—The mean of the measured altitudes should first be corrected for index error, if the sextant has been used, and also if the transit has been used without either reversing or using a mercury horizon. If the observations have been taken with the transit, half with the telescope direct and half with the telescope inverted, or with the transit in connection with a mercury horizon, no correction for index error is required. The resulting altitude should be further corrected for refraction and parallax, and, if the observations are made on a single edge of the sun, for semi-diameter.

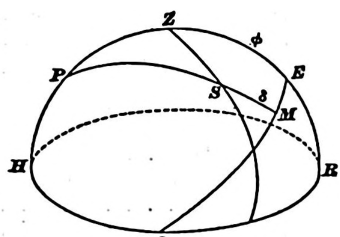

FIG. 4

24. Formula for Finding the Sun's Hour Angle From the True Altitude.—Let Fig. 4 represent a portion of the celestial sphere; let Z be the zenith of the observer; P, the celestial pole; HOR, the horizon; and OME, the equator. If S is the sun, the angle ZPS is the required hour angle; SM, the declination of the sun; PS, its polar distance; and ZS, its zenith distance.

If φ is the latitude of the observer, then $PZ = 90° - \varphi$ (*Practical Astronomy*, Part 1).

Let z = sun's zenith distance = ZS;
t = sun's hour angle = apparent solar time = ZPS;
δ = sun's declination = SM;
$m = \varphi - \delta$ = latitude − declination.

Then, the hour angle t is determined by means of the following formula:*

$$\sin \tfrac{1}{2} t = \sqrt{\frac{\sin \frac{1}{2}(z+m) \sin \frac{1}{2}(z-m)}{\cos \varphi \cos \delta}}$$

The value of t is $-$ in the morning and $+$ in the afternoon. To find the mean time and the error of the watch, the equation of time is added to the apparent solar time, the result being the local mean time of the observation. The difference between this result and the mean of the recorded times is the error of the watch on local time.

EXAMPLE 1.—The altitude of the lower edge of the sun was measured at Philadelphia on the afternoon of January 10, 1903. The instrument used was a sextant of which the index error was $+2' 40''$. The circle reading was $38° 10' 0''$; the corresponding watch time was $2^h 30^m 56^s$. The latitude of Philadelphia being $+39° 58'$ ($= \varphi$), the equation of time $+7^m 27.7^s$, and the declination of the sun $-22° 1' 39''$ ($= \delta$), to find the error of the watch.

SOLUTION.—To find the true altitude:

Observed double altitude	38° 10′ 00″
Index error	+2 40
Double altitude	38° 12′ 40″
Single altitude	19° 6′ 20″
Refraction (Table VII, *Practical Astronomy*, Part 1)	−2 43
Parallax (Table VIII, *Practical Astronomy*, Part 1)	+8
Semi-diameter (Table IV, *Practical Astronomy*, Part 1)	+16 18
True altitude of sun's center	19° 20′ 3″

To find the hour angle:

$$z = 90° - 19° 20' 3'' = 70° 39' 57''$$

$$m = 39° 58' - (-22° 1' 38.9'') = 61° 59' 39''$$

$$\tfrac{1}{2}(z+m) = \frac{70° 39' 57'' + 61° 59' 39''}{2} = 66° 19' 48''$$

$$\tfrac{1}{2}(z-m) = \frac{70° 39' 57'' - 61° 59' 39''}{2} = 4° 20' 9''$$

*This, and a few similar formulas used in this Section, are derived from the principles of spherical trigonometry, which the student may find in any work on the subject (Wood's "Plane and Spherical Trigonometry" is recommended for its conciseness). Spherical trigonometry is not included in this Course, as the applications of it that the Course contains are too few to justify a complete treatment of that subject, which is comparatively complicated.

Substituting the given values in the formula,

$$\sin \tfrac{1}{2} t = \sqrt{\frac{\sin 66° 19' 48'' \sin 4° 20' 9''}{\cos 39° 58' \cos 22° 1' 39''}}; \ \tfrac{1}{2} t = 18° 11' 28''; \ t = 36° 22' 56''.$$

Dividing by 15 to reduce to time measure, we find $2^h 25^m 32^s$ P. M. as the apparent solar time.

To find the watch error:

Apparent solar time	$2^h 25^m 32.0^s$
Equation of time	$+7 \ 27.7$
Local mean time	$2^h 32^m 59.7^s$
Watch time	$2 \ 30 \ 56.0$
Hence, the watch was slow	$2^m 3.7^s$

on local time, and the watch error was $-2^m 3.7^s$. Ans.

EXAMPLE 2.—For the purpose of illustrating a complete series, both on the upper and on the lower edge of the sun, we shall take the following data from a survey made in Colorado. The observations were taken with an engineer's transit, as explained in Art. **21**. The index error was $-28''$; the latitude, $+38° 04'$; the longitude, $+1^h 44^m 41^s$; the declination of the sun, $+18° 42' 17''$; and the equation of time, $+6^m 13^s$. To find the error of the chronometer.

Edge of Sun	Circle Readings			Time		
Upper	44°	25′	00″	8^h	35^m	12.0^s
	44	30	0		35	39.5
	44	35	0		36	3.5
	44	40	0		36	30.5
	44	45	0		36	56.5
Lower	44	25	0	8	37	55.5
	44	30	0		38	22.0
	44	35	0		38	48.0
	44	40	0		39	14.5
	44	45	0		39	41.0
Means	44	35	0	8	37	26.3

SOLUTION.—

Mean circle reading	44° 35′ 0″
Correction for index error	−28
Measured altitude	44° 34′ 32″
Refraction (− 58″) and parallax (+ 6″)	−52
True altitude	44° 33′ 40″

No correction for semi-diameter.

To find the hour angle:

$$m = 38° \, 4' \, 0'' - 18° \, 42' \, 17'' = 19° \, 21' \, 43''$$

$$z = 90° - 44° \, 33' \, 40'' = 45° \, 26' \, 20''$$

$$\tfrac{1}{2}(z + m) = \frac{45° \, 26' \, 20'' + 19° \, 21' \, 43''}{2} = 32° \, 24' \, 2''$$

$$\tfrac{1}{2}(z - m) = \frac{45° \, 26' \, 20'' - 19° \, 21' \, 43''}{2} = 13° \, 2' \, 19''$$

Substituting the given values in the formula,

$$\sin \tfrac{1}{2} t = \sqrt{\frac{\sin 32° \, 24' \, 2'' \sin 13° \, 2' \, 19''}{\cos 38° \, 4' \cos 18° \, 42' \, 17''}}$$

$\tfrac{1}{2} t = 23° \, 44' \, 33''$; $t = 47° \, 29' \, 6''$. Dividing by 15 to reduce to time measure, $t = 3^h \, 9^m \, 56.4^s$. The hour angle is minus in the morning. The apparent solar time is found by subtracting $3^h \, 9^m \, 56.4^s$ from 12 hr., which gives $8^h \, 50^m \, 3.6^s$ A. M. apparent solar time.

To find the chronometer error:

Apparent solar time	$8^h \, 50^m \; 3.6^s$
Equation of time	$+6 \; 13.0$
Local mean time	$8^h \, 56^m \; 16.6^s$
Mean of the ten recorded times	8 37 26.3
Hence, the chronometer was slow	$18^m \; 50.3^s$

and its error was $-18^m \, 50.3^s$. Ans.

EXAMPLES FOR PRACTICE

1. Ten altitudes of the sun were measured with a transit as in Art. **21**, one-half of them being on the lower and one-half on the upper edge of the sun. The transit had a complete vertical circle, and the index error was eliminated. The mean of the ten measured altitudes was 28° 30′ 44″; the mean of the ten corresponding times was $2^h \, 5^m \, 45.5^s$ P. M. The declination of the sun being −12° 8′ 16″, the equation of time being $-15^m \, 50.5^s$, and the latitude being +39° 57′ 6″, find the error of the chronometer. Ans. $+33.5^s$

2. Ten double altitudes of the sun were measured with the sextant, one-half being on the upper and one-half on the lower edge. The mean circle reading (double altitude) was 77° 40′, and the mean of the corresponding times was $9^h \, 53^m \, 19.3^s$ A. M. The index correction being −15′ 50″; the declination, −3° 2′ 26″; the latitude, +40° 36′ 24″; and the equation of time, $-10^m \, 11.5^s$, find the chronometer error.

Ans. $+1^m \, 41.8^s$

SECOND METHOD OF DETERMINING TIME: FROM THE ALTITUDE OF A STAR

25. The altitude of a bright star when 3 or 4 hours from the meridian is measured as described in Art. 20; the resulting altitude is corrected for index error and refraction, and the corresponding hour angle is computed exactly as in Art. 24. To this hour angle is added the right ascension of the star, and the sum, which is the sidereal time, is changed into mean time and compared with the time as shown by the chronometer. The following example will fully illustrate the process:

EXAMPLE.—The altitude of the star α Coronæ Borealis was measured with a transit instrument having an incomplete circle, and with the aid of a mercury horizon, as explained in *Practical Astronomy*, Part 1. The observations were made as described in Art. 20, the resulting measurements being as follows:

Telescope Circle		Time
Direct	$+47° \; 45' \; 0''$	$10^h \; 2^m \; 56.0^s$
Reflected	$-47 \; 44 \; 40$	$4 \; 4.8$
Reflected	$-47 \; 44 \; 25$	$4 \; 26.5$
Direct	$+47 \; 44 \; 10$	$4 \; 57.5$

The right ascension of the star was $15^h \; 29^m \; 34.1^s$; the declination, $+27° \; 7' \; 32''$; the approximate latitude, $+38° \; 4'$. The sidereal time of mean noon was computed and found to be $8^h \; 21^m \; 15.7^s$. To find the error of the chronometer.

SOLUTION.—To find the true altitude:

Mean of the direct readings	$+47° \; 44' \; 35''$
Mean of the reflected readings	$-47 \; 44 \; 33$
Measured double altitude	$95° \; 29' \; 08''$
Measured altitude	$47° \; 44' \; 34''$
Refraction	-52
True altitude	$47° \; 43' \; 42''$

No correction for index error.

To find the star's hour angle:

$$m = +38° \; 4' - 27° \; 7' \; 32'' = 10° \; 56' \; 28''$$
$$z = 90° - 47° \; 43' \; 42'' = 42° \; 16' \; 18''$$

$$\tfrac{1}{2}(z+m)=\frac{42^\circ\ 16'\ 18''+10^\circ\ 56'\ 28''}{2}=26^\circ\ 36'\ 23''$$

$$\tfrac{1}{2}(z-m)=\frac{42^\circ\ 16'\ 18''-10^\circ\ 56'\ 28''}{2}=15^\circ\ 39'\ 55''$$

$$\sin\tfrac{1}{2}t=\sqrt{\frac{\sin 26^\circ\ 36'\ 23''\ \sin 15^\circ\ 39'\ 55''}{\cos 38^\circ\ 4'\ \cos 27^\circ\ 7'\ 32''}}\ ;\ \tfrac{1}{2}t=24^\circ\ 32'\ 47''$$

$t = 49^\circ\ 5'\ 34''$. Dividing by 15 to reduce to time measure, $t = 3^h\ 16^m\ 22^s$.

To find the sidereal time, t is added to the right ascension $15^h\ 29^m\ 34.1^s$; therefore, the sidereal time $= 18^h\ 45^m\ 56.1^s$.

To find the mean time:

Sidereal time of mean noon	$8^h\ 21^m\ 15.7^s$
Sidereal interval past mean noon	10 24 40.4
Correction, Table II of Ephemeris	−1 42.3
Mean solar time	$10^h\ 22^m\ 57.1^s$
Chronometer time = mean of above four recorded times	10 4 6.2
Hence, the clock was slow	$18^m\ 50.9^s$

This observation was made in the evening of the same day as that of the second example of Art. 24, in order to verify the result found in the morning. When this is done, the observation in the afternoon or evening should be made on a body whose altitude is, as nearly as possible, equal to that of the body observed in the morning. Errors of the vertical circle readings and of adjustment are thus largely eliminated.

DETERMINATION OF LONGITUDE

26. By a Chronometer.—The difference in longitude between two points is simply the difference between their local times. Hence, when the error of the watch or chronometer from the local time of one station has been determined by one of the preceding methods, it is only necessary to compare it with the error from local time at a second station whose longitude is known, to find the difference in longitude between the two stations.

Between two fixed points of observation the method is as follows: One or more chronometers are compared with the standard clock at the first place of observation; they are then carried to the second place and compared with the standard clock there; finally, they are brought back to the starting place and a third comparison is made. If the errors of the two clocks on the local mean times are well determined, and

also the rate at which the first clock is gaining or losing time, the longitude can thus be determined with much accuracy.

Let the two points of observation be denoted by A and B. Let O_1 and C_1 be two corresponding times of the observatory clock and the chronometer, respectively, at A; that is, let C_1 be the time shown by the chronometer when the time indicated by the clock, after the clock error has been applied, is O_1. Let O_2 and C_2 be corresponding times at B, when the chronometer is taken to that station and compared with the clock. Finally, let O_3 and C_3 be corresponding times of the clock and chronometer at A, when the chronometer is brought back to A. Then, the difference d in longitude between the two stations is given by the formula

$$d = (C_2 - O_2) - (C_1 - O_1) - [(C_3 - O_3) - (C_1 - O_1)]\frac{C_2 - C_1}{O_3 - O_1}$$

In applying this formula, it should be borne in mind that $C_2 - C_1$ denotes the interval elapsed between the first and the second comparison, and $O_3 - O_1$ (for which $C_3 - C_1$ may be substituted without any serious error) denotes the time elapsed between the first and the third comparison.

EXAMPLE.—On the morning of May 28, a chronometer was compared with the observatory clock at Philadelphia. It was then carried to Washington and compared with the standard clock of the Naval Observatory, and on the morning of the next day was brought back and compared with the Philadelphia clock. From a long series of observations, it was known that the Philadelphia clock was at that time 38.42^s fast, and was gaining 1.12^s each 24 hours; the Washington clock was 8.60^s slow. To find the longitude of Philadelphia from Washington, the comparisons being as follows:

Place	Observatory Clock	Chronometer
Philadelphia	May 28, 11^h 14^m 20^s	11^h 36^m 24.22^s
Washington	May 28, 21 17 00	21 47 11.64
Philadelphia	May 29, 9 58 00	10 20 4.85

SOLUTION.—First the clock times are corrected by applying the errors and rates given above.

First time	11^h 14^m 20.00^s
Error	−38.42
O_1	11^h 13^m 41.58^s

Second time	$21^h\ 17^m\ 00.0^s$
Error	$+8.6$
O_2	$21^h\ 17^m\ 8.6^s$
Third time	$9^h\ 58^m\ 00.0^s$
$-\left(38.42+\frac{22.8}{24}\times 1.12^s\right)$	-39.48
O_3	$9^h\ 57^m\ 20.52^s$

The fraction $\frac{22.8}{24}\times 1.12^s$ is the gain of the clock between the first and third times, 22.8 being the number of hours elapsed between them. Thus, O_1, O_2, and O_3 are the absolutely correct times at the instants of comparison; that is, the times the clocks would have shown were they wholly free from error.

To apply the formula given in Art. 26, we have

$C_2 - O_2 = 30^m\ 3.04^s$; $C_1 - O_1 = 22^m\ 42.64^s$

$C_3 - O_3 = 22^m\ 44.33^s$; $C_2 - C_1 = 10^h\ 10^m\ 47.42^s = 10.180^h$

$O_3 - O_1 = 22^h\ 43^m\ 38.94^s = 22.728^h$

Substituting these values in the formula,

$$d = 30^m 3.04^s - 22^m 42.64^s - (22^m 44.33^s - 22^m 42.64^s)\frac{10.180}{22.727} = 7^m\ 19.64^s$$

Ans.

27. Method of Comparing a Watch or Chronometer With a Standard Clock.—It is important that the student know how to find accurately to $\frac{1}{10}$ second, or closer, the difference between the times shown by his watch and by the clock with which he compares it. The method is very simple; it consists in noting the watch time of chosen clock beats, and may be thus stated:

Rule.—*Look at the clock and begin to count the beats; still counting each beat, look at the watch and note the watch time estimated to tenths of a second when a chosen clock second is beaten. Repeat the observation five or ten times, and find the mean of the difference between the corresponding times. If the work has been carefully done, this mean should not be uncertain by more than $\frac{1}{10}$ of a second.*

For example, if we wish to record the watch time corresponding to $9^h\ 34^m\ 30^s$, we look at the clock at about the twentieth second, and then, looking at the watch and counting each second as we hear the clock beat it, we note the exact watch instant when the thirtieth second is beaten. We

write down the seconds and tenths of a second from the watch, and add the hours and minutes immediately afterwards.

Thus, in comparing a watch with the observatory clock, the following five clock times were first written and then it was observed that when the clock beat $8^h\ 20^m\ 10^s$, the watch showed $8^h\ 20^m\ 16.4^s$, and similarly with the others:

Clock	Watch	Difference
$8^h\ 20^m\ 10^s$	$8^h\ 20^m\ 16.4^s$	6.4^s
8 20 30	8 20 36.5	6.5
8 20 50	8 20 56.5	6.5
8 21 10	8 21 16.4	6.4
8 21 30	8 21 36.4	6.4

The mean of the difference is 6.44^s. The clock on this date was 12.64^s fast, and therefore the watch was $6.44 + 12.64 = 19.1^s$ fast, which is probably correct within $\frac{1}{10}$ second.

28. At nearly all telegraph offices in the United States, the standard time is now received, either from Washington or from a nearer observatory, at exactly standard noon each day. At many seaports, time balls or other time signals also serve to show the navigator and others on each day the instant of standard noon. If the observer has but a single watch or chronometer, whose error on local mean time he determines by his observations, he must in some way find its *daily rate;* that is, the amount that the watch or chronometer gains or loses in 24 hours.

29. Daily Rate.—To find the daily rate, the error at two different times is determined either by observation or by comparison with a clock whose error is known. The difference between the two errors divided by the number of days and decimals of a day elapsed between the two determinations is the **daily rate.**

EXAMPLE.—At station *A*, on July 2, 9 A. M., a watch was found, by the method of Art. 22, to be $1^m\ 18.1^s$ fast. On July 4, 3 P. M., it was

found to be $1^m\ 34.2^s$ fast at station *A*, and on July 5, at 12 noon, comparing it with the standard 75th-meridian time at a telegraph office in Philadelphia, it was found to be 43.4^s fast. To find the longitude of station *A* from Washington.

SOLUTION.—The rate of the chronometer is first found. From July 2, 9 A. M., to July 4, 3 P. M., is 2 da. 6 hr. = 2.25 da. Since in 2.25 da. the watch gained 16.1 sec., the daily rate was $16.1^s \div 2.25 = 7.15^s$. (This is a very large rate.)

Next, the last chronometer time must be freed from the effect of rate. From July 2, 9 A. M., to July 5, 12 M., is 3 da. 3 hr. = 3.125 da. The gain during this time was $3.125 \times 7.15^s = 22.34^s$. Hence, the last reading freed from rate is $43.4^s - 22.3^s = 21.1^s$ fast.

Thus, the chronometer was $1^m\ 18.1^s$ fast on station *A* local time, and 21.1^s fast on 75th-meridian time. Hence, station *A* is $1^m\ 18.1^s - 21.1^s = 57.0^s$ west of the standard meridian; that is, $5^h\ 0^m\ 57^s$ west of Greenwich. Since Washington is $5^h\ 8^m\ 15.78^s$ west of Greenwich (Ephemeris), it follows that station *A* is $7^m\ 18.78^s$ east of Washington. Ans.

30. Use of Several Chronometers.—It has so far been assumed that the observer had only one accurate watch or chronometer. If a standard clock the error and rate of which are accurately known is not easily accessible, the observer must make observations at frequent intervals (preferably once each day) in order to keep informed regarding the rate of his chronometer. This assumes that he remains for several days at one station. When several stations are occupied successively and each for a short time, and also on sea voyages, it is necessary to carry three chronometers whose rates and whose errors on Greenwich or Washington mean time have been well determined before starting and which will be again examined immediately after the return. It is necessary to have three chronometers, since, if there were but two, there would be no way of telling which of the two was in error should one of them begin suddenly to gain or to lose.

DETERMINATION OF AZIMUTH

INTRODUCTION

31. The line in which the plane of the meridian intersects the plane of the horizon is called the **north-and-south line**, or the **true meridian.** It follows that the north-and-south line passes through the north and the south point of the horizon.

32. As explained in *Compass Surveying*, Part 1, the **true azimuth** of a line on the surface of the earth is the angle that the line forms with the true meridian. In astronomy, the term azimuth is always understood to mean true azimuth.

In what follows, azimuth will be reckoned from the north toward the east, unless otherwise stated. A negative azimuth is an azimuth reckoned in the opposite direction from that in which positive azimuths are reckoned. Thus, if positive azimuths are reckoned from the north toward the east, the azimuth of PG, Fig. 5, is either $+NESG$ or $-NAG$.

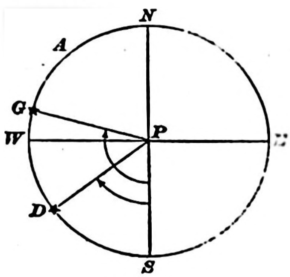

FIG. 5

33. Methods of Determining Azimuth.—The following are the most important methods of finding the azimuth of a line by astronomical observations with the engineers' transit: (1) by observing the sun directly at any hour angle; (2) by observing the sun with the aid of a solar transit; (3) by observing Polaris at eastern or western elongation; (4) by observing a south star at any hour angle; (5) by observing Polaris at any hour angle.

All these methods of determining azimuth are valuable, and when applied under proper conditions give good results; but the third, fourth, and fifth are the most satisfactory.

Observations of Polaris at eastern or western elongation afford at once the simplest and most accurate method of determining azimuth, but this method possesses the single disadvantage that it sometimes requires the observations to be made at an inconvenient hour. When this is the case, the fourth or fifth method can be substituted advantageously. The method by observing Polaris at culmination, described in *Transit Surveying*, Part 2, is rather unsatisfactory, as it requires that the observer's watch keep absolutely correct time, and that the observation be made at exactly the proper instant.

FIRST METHOD OF DETERMINING AZIMUTH: BY THE SUN AT ANY HOUR ANGLE

34. Outline of Method.—Let Fig. 6 represent the celestial sphere, *N E S W* being the horizon plane and *N Z S* the plane of the meridian. Let the observer be at *O*, and let the straight line joining the observer with some fixed mark or reference point in the horizon plane have the direction *O M*. It is this line whose azimuth *N O M* is to be determined.

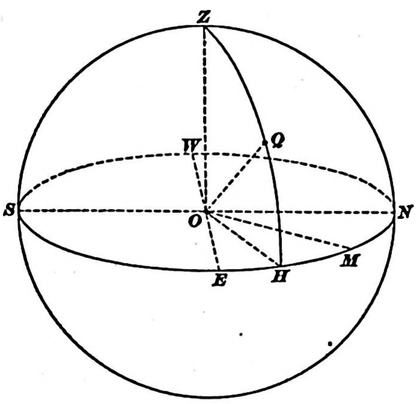

Fig. 6

Let *Q* be the position of the sun on the celestial sphere. Then *N O H* is the azimuth of the sun, and *Q O H* its altitude. When the altitude *Q O H* has been measured, the azimuth *N O H*, or *N H*, can be computed by trigonometry. The horizontal angle *M O H*, between the reference mark and the sun is measured, always turning the instrument toward the right, or in a clockwise direction. The azimuth of the line is determined by subtracting the horizontal angle *M O H* from the azimuth *N O H* of the sun.

35. Method of Making the Observations.—The transit must be provided with a vertical circle, and also with a colored-glass cap to protect the eye. The adjustments of the levels must be made with great care, especially those of the plate level parallel to the vertical circle; the index error must also be well determined, and both the vertical and horizontal wires must be in good adjustment.

The instrument is set up over the station *P*, Fig. 7, and the vernier of the horizontal circle set to read zero. The telescope is then directed to a flag at *M*, care being taken to make the final bisection by turning the lower tangent screw, so that the setting of the vernier will not be altered.

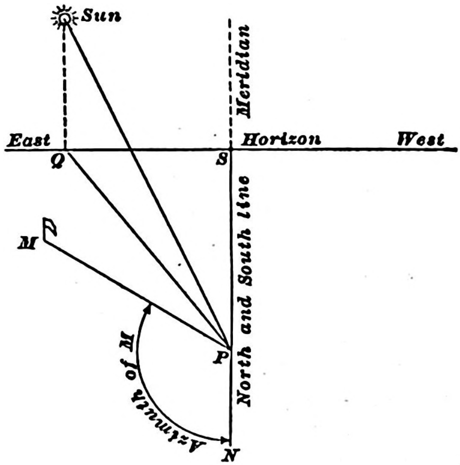

FIG. 7

The upper plate is now unclamped and the telescope is directed to the sun. After making this pointing as described in the next article, the readings of both the horizontal and vertical circle are recorded. This completes a single observation. It is customary to take five or six such sightings on the sun, the five or six corresponding readings of the horizontal and vertical circle being recorded. The lower plate is kept constantly clamped throughout the series of observations. A sighting is finally taken on the azimuth mark *M*; if the reading differs appreciably from zero, this shows that the plates have slipped, and the observations must be rejected. The two sightings on the mark with the five or six pointings on the sun constitute a series.

36. Sighting at the Sun.—The angular diameter of the sun is about 0° 32′. In the high-power telescopes now

generally used on transits, such an image will largely fill up the field of view. It is impossible to bisect such a large image with sufficient accuracy, and, consequently, when the telescope is pointed to the sun, the intersection of the cross-wires cannot be placed accurately at the center of the sun. It is necessary to place the horizontal and vertical wires tangent to the apparent disk, Fig. 8, and to apply to the readings on both the horizontal and the vertical circle a correction for semi-diameter. The novice will be surprised at the rapidity of the sun's motion when viewed through a high-power telescope. In the forenoon, the sun is moving upwards and to the right. For morning work, therefore, the telescope is set so that the upper edge of the sun is somewhat below the middle cross-wire and the right-hand edge is somewhat to the left of the vertical wire. If the telescope has stadia wires, care must be taken that neither the upper nor the lower stadia wire is mistaken for the middle wire. Then, with one hand on the tangent screw to the vertical arc and the other hand on the tangent screw to the horizontal plate, either or both screws are adjusted so as to make the sun reach both cross-wires at the same instant. Some practice will be required in order to do this accurately. In the afternoon, the sun is moving downwards and to the right, and the telescope should be so set that the sun will move into the upper left-hand corner.

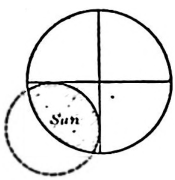

FIG. 8

37. Corrections.—The measured altitude of the sun's edge must be corrected for index error, refraction, and semi-diameter. The reading of the horizontal circle must be corrected for semi-diameter. The correction to the vertical circle reading is simply the angular semi-diameter of the sun, and this should be subtracted from the reading if the upper edge of the sun has been observed, but added to the reading if the wire has been placed tangent to the lower edge of the sun.

The correction to the horizontal reading is effected in the following manner: Let O, Fig. 9, be the center of the celestial

sphere, Z the zenith, and WHK the horizon; let the telescope be at O and directed to the sun's edge at T, and let ST be the radius of the sun.

When the intersection of the cross-wires is moved from T to S, the horizontal angle KOH is turned off on the horizontal circle; it is desired to find this horizontal angle, or what is the same thing, the angular length of the arc KH, which is the correction for semi-diameter to be applied to the horizontal circle reading.

The angles TRS and KOH are equal, since each is equal to the spherical angle KZH (*Practical Astronomy*, Part 1). Hence, $\frac{\text{arc } ST}{\text{arc } HK} = \frac{RS}{OH} = \frac{RS}{OS}$; for, as shown in geometry, the circumferences of any two circles are to each other as their radii, and arcs subtending equal angles are to each other as their radii.

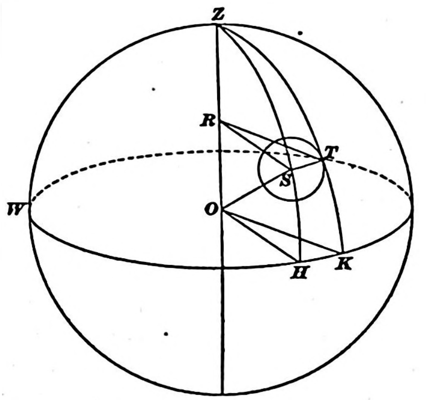

FIG. 9

But the angle SOH is the sun's altitude, and, consequently, the equal angle RSO also equals the sun's altitude. In the triangle RSO, $\frac{RS}{OS} = \cos RSO =$ cosine of sun's altitude.

Hence, cosine of sun's altitude $= \frac{\text{arc } ST}{\text{arc } HK}$, or arc HK = arc $ST \div$ cosine of sun's altitude.

Hence, the correction to be applied to the reading of the horizontal circle is found by dividing the sun's semi-diameter by the cosine of its altitude. This correction is to be added to the reading of the horizontal circle if the wire is placed tangent to the left edge of the sun, but subtracted from the reading of the horizontal circle if the wire is placed tangent to the right edge of the sun.

The semi-diameter of the sun may be taken with sufficient accuracy from the following table, which is given in *Practical Astronomy*, Part 1, and is here repeated for convenience:

Time of Year (Approximately)	Semi-Diameter of Sun	Time of Year (Approximately)	Semi-Diameter of Sun
January 1 . .	16′ 18″	July 1	15′ 45″
April 1	16 2	October 1 . .	16 2

38. Formula for Finding the Sun's Azimuth When the True Altitude is Known.

Let z = zenith distance of the sun;
t = hour angle;
δ = declination;
φ = latitude of observer;
a = azimuth, counted from north toward east.

The value of a is computed by the following formula, which is derived by the principles of spherical trigonometry:

$$\sin \tfrac{1}{2} a = \sqrt{\frac{\cos \frac{1}{2} (z + \varphi + \delta) \sin \frac{1}{2} (z + \varphi - \delta)}{\sin z \cos \varphi}}$$

In applying this formula, it should be remembered that there are always two angles corresponding to a given sine. Thus, if $\sin A = .5$, the angle A may be either 30° or 150° (= 180° − 30°), since any two supplementary angles have the same sine. This is true in all cases, the formula $\sin (180° - A) = \sin A$ being perfectly general, whether A is greater or less than 180°. For example, $\sin 275° = \sin (180° - 275°) = \sin (-95°) = -\sin 95° = -\sin (180° - 95°) = -\sin 85°$.

As explained in Art. 32, a negative azimuth equal to $-a$ is the same as a positive azimuth equal to $360° - a$.

It will be observed that, in the forenoon, the sun is east of the meridian, and its azimuth is, therefore, less than 180°. In the afternoon, the sun is west of the meridian, and its azimuth is greater than 180°. Of the two values of $\frac{1}{2} a$ given by the formula, one is acute and the other obtuse. For morning observations, the acute angle should be used; for afternoon observations, the obtuse.

The zenith distance z is obtained by subtracting the corrected altitude from 90°. The value of φ must be known at least approximately.

The desired azimuth of the line is obtained by subtracting the corrected mean reading of the horizontal circle from the azimuth of the sun. If the minuend of this subtraction is less than the subtrahend, it should be increased by 360°. Let the student verify this by drawing a sketch showing the object M, Fig. 7, on the right of PS.

It must be remembered that the horizontal angles are measured in a clockwise direction.

EXAMPLE.—The following observations were taken in the morning, in the manner described in Art. 35:

Vertical Circle	Horizontal Circle	Diagram of Field
22° 48.5′	237° 41.0′	
22 12.5	238 11.0	
21 44.5	238 34.0	
21 19.0	238 55.0	
20 49.5	239 19.5	
20 28.0	239 38.0	

The declination of the sun was +14° 45′ 40″, the approximate latitude was 39° 58′, and the semi-diameter of the sun was 15′ 54″; the index error was eliminated. To find the azimuth of the sun.

SOLUTION.—

Mean of the vertical circle readings	21° 33′ 40″
Refraction (Table VII, *Practical Astronomy*, Part 1)	−2 24
Parallax (Table VIII, *Practical Astronomy*, Part 1)	+8
Semi-diameter, which is to be subtracted (see field diagram)	−15 54
True altitude	21° 15′ 30″
Zenith distance = 90° − true altitude	68 44 30
Mean reading of horizontal circle	238° 43′ 5″
Correction for semi-diameter = 15′ 54″ ÷ cos 21° 15′ 30″ = 954″ ÷ 0,932, to be subtracted (see field diagram)	−17 4
True horizontal angle to the center of the sun .	238° 26′ 1″

To find the azimuth of the sun:

$$z = 68° 44' 30''; \quad \varphi = 39° 58'' 0''; \quad \delta = 14° 45' 40''$$

$$\tfrac{1}{2}(z + \varphi + \delta) = 61° 44' 5''; \quad \tfrac{1}{2}(z + \varphi - \delta) = 46° 58' 25''$$

Substituting these values in the formula,

$$\sin \tfrac{1}{2} a = \sqrt{\frac{\cos 61° 44' 5'' \sin 46° 58' 25''}{\sin 68° 44' 30'' \cos 39° 58'}}$$

$$\tfrac{1}{2} a = 44° 7' 14''; \quad a = 88° 14' 28''$$

The acute angle corresponding to $\sin \frac{1}{2} a$ is taken, because the observation was made in the forenoon. As the horizontal reading 238° 26′ 01″ is greater than the computed azimuth of the sun, 360° should be added to the latter, before subtracting the former. Then, the azimuth of the mark is

$$360° + 88° 14' 28'' - 238° 26' 1'' = 209° 48' 27''. \quad \text{Ans.}$$

EXAMPLE FOR PRACTICE

The following observations were taken in the manner described in Art. 35:

Approximate Time A. M.	Vertical Circle	Horizontal Circle	Diagram of Field
$8^h\ 40^m$	43° 09′ 0″	64° 42′ 0″	
8 42	43 35 30	65 10 30	

The declination of the sun was +19° 43′ 10″; the latitude was +40° 36′ 27″; and the semi-diameter was 15′ 49″. Find the azimuth.

Ans. 36° 34′ 31″

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39. In this method, each single observation may be reduced separately, if desired. When this is done, the agreement of the individual values of the azimuth will furnish a test of the accuracy with which the observer has made the observations, but not a test of the accuracy of the result, since the errors of adjustment which always produce a comparatively large error in all determinations of azimuth, have not been eliminated. The additional labor of reducing each observation separately is considerable, and is not justified by the character of the work done. It would be much better to devote the additional time to taking another series; the two series may then each be reduced as in the preceding article

If the vertical circle of the transit is not complete, the best determination of azimuth will be secured by taking several series of observations. If the vertical circle is complete, at least two series of observations should be taken, one with the telescope direct and one with it reversed, if this method is used at all. When the transit has a complete vertical circle, the following modification of the method is preferred:

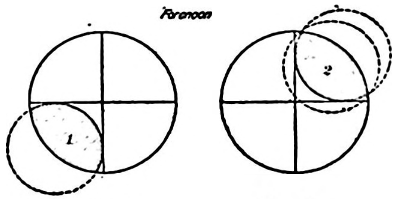


Fig. 10 Fig. 11

40. Modification of the Method When the Vertical Circle is Complete.—In this modification, the errors of adjustment, the index error, and the corrections for semi-diameter are eliminated by the method of taking the observations. The computation is performed exactly as in Art. 38, except that the corrections just referred to are omitted.

The instrument is set up as before, with the horizontal plate reading 0° when sighting at the azimuth mark. For forenoon work, the sun should be so sighted that it occupies position *1*, Fig. 10, with reference to the cross-wires. The time, vertical angle, and horizontal angle are noted as before. Then the upper plate is loosened, the instrument turned 180° in azimuth, the telescope inverted, and the sun sighted again, as in position *2*, Fig. 11. In position *1*, Fig. 10, the sun is moving toward both wires; in position *2*, Fig. 11, the telescope should be set approximately as shown by the dotted circle, so that the sun will clear both wires at the same instant. For afternoon work, the positions shown in Figs. 12 and 13 should be used. The observations are taken in *pairs;* if the second observation of a pair cannot be obtained promptly after the

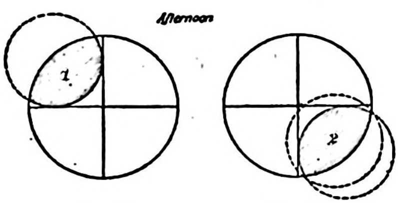


Fig. 12 Fig. 13

first one (owing to a passing cloud, or some other cause), the first must be ignored and considered as useless.

41. It should be noted that the reversal of the transit between the observations eliminates the index error of the vertical circle, the error of level in the horizontal axis of the telescope, and the error of collimation of the telescope. By sighting in diagonal corners of the field of view and taking the mean of the observations, the corrections (both horizontal and vertical) due to the semi-diameter of the sun are eliminated. To simplify the notes, 180° should be added to (or subtracted from) the horizontal plate reading when the instrument is inverted.

EXAMPLE.—The following measurements were taken in the manner described in this article. The four means of the circle readings were formed in the field. The declination of the sun was −9° 30′ 5″, and the approximate latitude +39° 57′. To find the azimuth.

Telescope	Time P. M.	Vertical Circle	Horizontal Circle
Direct . . .	3:27	19° 39′ 00″	99° 52′ 00″
Inverted . .	3:29	19 52 00	99 49 00
Mean . . .	3:28	19 45 30	99 50 30
Direct . . .	3:32	18 46 00	100 55 30
Inverted . .	3:34	19 3 00	100 49 00
Mean . . .	3:33	18 54 30	100 52 15
Direct . . .	3:36	18 4 30	101 46 00
Inverted . .	3:38	18 23 30	101 35 00
Mean . . .	3:37	18 14 00	101 40 30
Direct . . .	3:40	17 26 30	102 29 30
Inverted . .	3:42	17 43 00	102 21 00
Mean . . .	3:41	17 34 45	102 25 15

SOLUTION.—

Mean of the four vertical circle readings	18° 37′ 11″
Refraction (Table VII, *Practical Astronomy*, Part 1)	−2 48
Parallax (Table VII, *Practical Astronomy*, Part 1)	+8
True altitude of center	18° 34′ 31″
Zenith distance = 90° − true altitude	71° 25′ 29″

To find the azimuth of the sun: $z = 71° 25' 29''$; $\varphi = 39° 57' 0''$; $\delta = -9° 30' 5''$; $\frac{1}{2}(z + \varphi + \delta) = 50° 56' 12''$; $\frac{1}{2}(z + \varphi - \delta) = 60° 26' 17''$.
Substituting these values in the formula of Art. 38,

$$\sin \tfrac{1}{2} a = \sqrt{\frac{\cos 50° 56' 12'' \sin 60° 26' 17''}{\sin 71° 25' 29'' \cos 39° 57'}}$$

The two values of $\frac{1}{2} a$ are $60° 17' 15''$ and $119° 42' 45''$ ($= 180° - 60° 17' 15''$). As the observations were made in the afternoon, the obtuse angle should be used (see Art. 38). This gives

$$a = 2 \times 119° 42' 45'' = 239° 25' 30''$$

The mean of the four horizontal readings is $101° 12' 8''$. Subtracting this from the azimuth of the sun, the azimuth of the line is found to be $239° 25' 30'' - 101° 12' 8''$, or $138° 13' 22''$. Ans.

SECOND METHOD OF DETERMINING AZIMUTH: BY THE SOLAR ATTACHMENT

42. Description.—The foregoing methods have the advantage of requiring no extra equipment to the transit, except an inexpensive colored-glass cap, but they require considerable numerical computation. Some transits are provided with a device known as the **solar attachment,** by means of which the true meridian can be located directly from observation. There are several forms of solar attachment and of solar transit, but they all have the condition that when the latitude and declination have been properly set off and the instrument so turned that an image of the sun is formed in a specified place, the main telescope of the transit will lie in the meridian. A detailed description is here given of one of the best and simplest forms of solar attachment, known as the **Saegmuller solar attachment.**

A view of the attachment alone is shown in Fig. 14. It consists of a small telescope $E O$, having a colored-glass cap over the object glass O, and a prismatic eyepiece E, which permits the observer to look into the side of the eyepiece instead of into its end, thus enabling him to make an observation with the telescope nearly vertical. A level bubble is on the top of the telescope. At the ends of the level bubble are two circular sights s and s', the latter being somewhat smaller than the former. When directing the telescope to the sun, the quickest method is so to turn it that the shadow of the

smaller sight s' will fall on the sight s, forming a ring of light around its edge. When this occurs, the image of the sun will be found within the field of view in the telescope. A clamp screw C secures the telescope axis to the standards S, and a tangent screw T permits slow motion. The standards and the telescope revolve about an axis, called the **polar axis**, hidden by the standard in Fig. 14. A clamp screw M and a slow-motion screw N permit the accurate adjustment of the

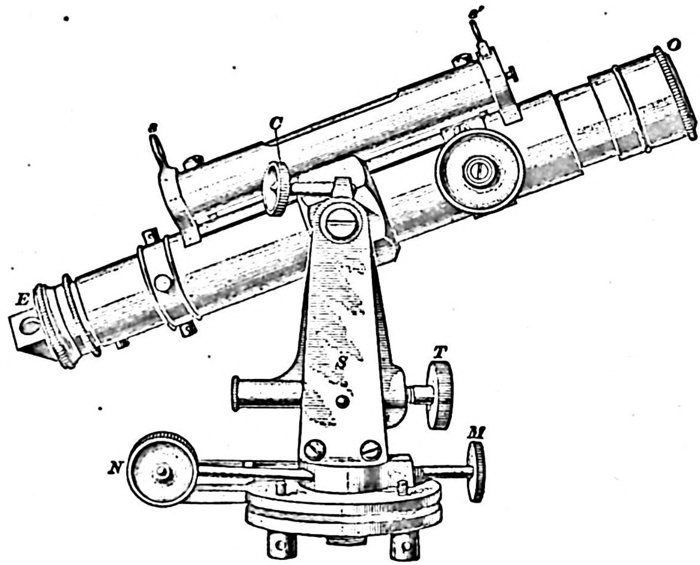


Fig. 14

instrument about its polar axis. By means of the screws in the base plate of the attachment, the polar axis is so adjusted as to be perpendicular to the line of collimation or optical axis, and also to the transverse axis of the transit telescope.

43. Preparation for Making the Observations. First, a table of declinations of the sun to be set off on the vertical circle is prepared. The declination of the sun for the beginning of each hour during the time in which the observations are to be made may be taken from the Ephemeris,

or from Table IV, *Practical Astronomy*, Part 1. These declinations are set off on the vertical circle of the transit in order to cause the solar telescope to point directly to the sun. As the apparent position of the sun in the sky is modified by refraction, a correction for refraction must be applied before setting off the declination on the vertical circle. If this were not done, the solar telescope would not point to the apparent position of the sun in the sky, but to its true position, and the sun would not be visible. This correction, which is to be added to the declination, may be taken directly from Table IV at the end of this Section, for any given latitude and hour angle.

EXAMPLE.—What declination should be set off on the vertical circle when the latitude is 39° 57′, the time 8:30 A. M., and the declination +3° 10′?

SOLUTION.—For 8:30 A. M. the hour angle is equal to $12 - 8.5 = 3.5$. Assume the latitude to be 40° (which is near enough); for declination 0° and hour angle 3, we have 1′ 9″, while for hour angle 4, we have 1′ 36″; hence, for hour angle 3.5 it should be the mean of these, or 1′ 22″. For declination +5°, we similarly compute 1′ 9″ for hour angle 3.5. The difference is $1' 22'' - 1' 9'' = 0' 13''$, and interpolating proportionally between the two values, for declination 3° 10′, we have $1' 22'' - \frac{3\frac{1}{6} \times 13}{5} = 1' 14''$. The declination to be set is, therefore, $3° 10' + 1' 14'' = 3° 11' 14''$. Ans.

44. Adjustment of the Instrument.—Adjust the eyepieces of both telescopes for parallax; focus the object glass of the solar for observing the sun; focus the object glass of the transit telescope for observing the azimuth mark. These adjustments should be made before observation is commenced, so as to avoid the jarring of the instrument that might occur if it were done later.

Level the instrument with extreme care. The plate bubble parallel to the telescope is almost invariably too sluggish for the accuracy required in this work. Therefore, clamp the telescope nearly horizontal; by repeated reversions of the whole instrument about its vertical axis, with corresponding adjustments of the leveling screws and the tangent screw to the vertical arc, the instrument may be so leveled that the

bubble of the telescope level will remain in the center of the tube for any position of the instrument. Under these conditions, the vertical axis is truly vertical and the reading of the vertical arc will be its index error, assuming that the line of collimation is parallel to the axis of the bubble tube. The exact error of the adjustment of the plate bubbles is then apparent and should be noted, so that any accidental change of level that may occur during the time of taking observations may be at once detected. The parallelism of the line of collimation and the axis of the telescope level can be tested and adjusted by the peg method described in *Leveling*.

45. Principle of the Instrument.—Set vernier A of the horizontal plates at 0° and point at the azimuth or reference mark. Loosen the upper plate and swing the telescope approximately into the meridian, with the objective end toward the south. If the declination is south, point the transit telescope upwards with a vertical angle equal to the value of the declination as modified by refraction. If the declination is north, point the telescope downwards with a vertical angle KOC, Fig. 15, equal to the modified declination. Then place the solar telescope in the same vertical plane as the transit telescope, make it horizontal by means of its level bubble, and clamp it securely, using screws C and T, Fig. 14.

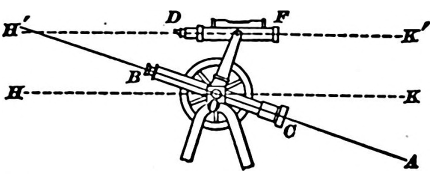


Fig. 15

Since the solar telescope is now horizontal, Fig. 15, and since the transit telescope is inclined to the horizon by an angle KOA equal to the sun's declination, it follows that the optical axis of the solar telescope and that of the transit telescope are inclined to each other at an angle $FH'C$, which is equal to the sun's declination.

Now bring the transit telescope into the plane of the equator. As the angle EOK, Fig. 16, between the plane of the equator and the plane of the horizon is equal to 90° minus

the latitude, this may be done by turning the transit telescope upwards until it makes a vertical angle with the plane of the horizon equal to 90° minus the latitude. The polar axis of the solar is thus made approximately parallel to the earth's axis *O P*, since it is perpendicular to the transit telescope *B C*, which now lies in the plane of the equator *O E*. It will be exactly parallel when the transit telescope is exactly in the meridian. The solar telescope is pointing above (or below) the plane of the equator by an angle *R H′ E*, equal to the sun's declination, since this is the angle between the axes of the two telescopes and the large telescope lies in the

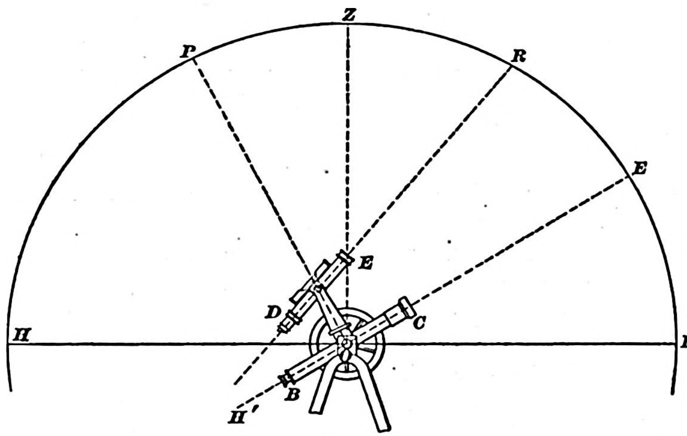


Fig. 16

equator plane. Hence, if we now turn the solar telescope about its polar axis, its line of collimation produced will trace out in the sky a small circle parallel to the equator. The declination of all points on this small circle will be equal to the sun's declination. Hence, the polar axis being in the plane of the meridian and parallel to the earth's axis, it follows that by turning the solar telescope about the polar axis it may be pointed directly to the sun. The converse is also true, that when the solar telescope is pointed directly to the sun, the polar axis will lie in the plane of the meridian. But since the polar axis and the optical axis of the transit

telescope are in the same vertical plane, it follows that when the polar axis lies in the plane of the meridian the optical axis of the transit telescope will lie in this plane also. Hence, when the solar telescope is pointed at the sun, the transit telescope will be in the plane of the meridian.

46. Method of Making the Observations.—Having adjusted the inclination of the two telescopes to the horizon, by setting off the proper angles on the vertical circle, as just explained, bring the transit telescope into the meridian as nearly as possible, and clamp the upper plate. Direct the solar telescope to the sun by the shadow of the sights on top of the solar telescope, and, with screw *M*, Fig. 14, clamp the motion about the polar axis. Then, with one hand on the slow-motion screw of the polar axis *N*, and the other hand on the tangent screw of the upper transit plate, point the solar telescope exactly at the center of the sun. When the instrument is in this position, the reading of the horizontal plate gives one value of the angle between the azimuth mark and the meridian. To obtain another value, which shall be independent of the previous determination, loosen the upper transit plate, the clamp screws *C* and *M* of the solar, and repeat the operation, making due allowance, if necessary, for any change of declination that may have taken place in the interval. With practice, several observations may be taken in a very few minutes, during which time no appreciable change of declination will take place, even when the motion of the sun in declination is most rapid. Watch the levels carefully for any indication of jarring or disturbance of the instrument. The lower plate should be kept clamped throughout, and the vernier should always read 0° when pointing at the azimuth mark. After completing a set of readings, check the reading on the azimuth mark to see if the plates have slipped. Observe the needle reading when the instrument is in the meridian; this will show the declination of the needle for that time and place. The mean of all the horizontal angles read is the desired azimuth.

EXAMPLE.—It is desired to make observations for azimuth with a solar transit at Philadelphia between 9 A. M. and 10 A. M. on

January 5, 1903. The latitude of Philadelphia is +39° 58′, and the longitude is $-7^m\ 37^s$. It is required to make out a table of corrected declination settings for use in the field.

SOLUTION.—First find the true declination of the sun for the hours of 9 A. M. and 10 A. M., Philadelphia mean time.

January 5, 9 A. M. civil time = January 4, 21^h, astronomical time. Longitude of Philadelphia (Ephemeris) is $-7^m\ 37^s = .127^h$. Washington time corresponding to 9 A. M. is January 4, 20.873^h. The declination (Table IV, *Practical Astronomy*, Part 1) for noon of January 4 at Washington is	−22° 47′ 42.8″
Hourly motion in declination is +15.06″. Multiplying this hourly motion by the number of hours that have elapsed between Washington noon and 9 A. M., we obtain the increase in the sun's declination during this period = +15.06″ × 20.873	+5′ 14.3″
Declination at 9 A. M., to the nearest second	−22° 42′ 28.0″
Add to the declination at 9 A. M. the hourly motion	+15
Declination at 10 A. M.	−22° 42′ 13″

We now form the following table:

Time	Declination	Correction for Refraction	Vertical-Circle Setting
$9^h\ 0^m$	−22° 42′ 28″	+3′ 2″	−22° 39′ 26″
9 10	−22 42 25	+2 54	−22 39 31
9 20	−22 42 23	+2 46	−22 39 37
9 30	−22 42 20	+2 38	−22 39 42
9 40	−22 42 18	+2 30	−22 39 48
9 50	−22 42 15	+2 22	−22 39 53
10 0	−22 42 13	+2 14	−22 39 59

The second column is found by simply adding one-sixth of 15.06″ = 2.51″, successively, to the declination, beginning with that for 9 A. M. The third column is computed from Table IV. As explained in Art. 43, corresponding to the latitude of 40° and an hour angle of 3 hours, we have

Correction for a declination of −20°	+2′ 36″
Correction for a declination of −23° 27′	+3 9
Difference	+33″

Hence, the correction corresponding to the given declination of $-22° 42'$ is found by interpolation to be

$$2' 36'' + \left(\frac{2° 42'}{3° 27'} \times 33''\right) = 2' 36'' + \left(\frac{270}{345} \times 33''\right) = 3' 2''.$$

Similarly, the correction for 10 A. M. is found to be $+2' 14''$, and the other numbers of the column are found by simply adding one-sixth of the difference between these two successively. Finally, by adding each correction to the corresponding number of the preceding column, we obtain the last column of settings for use in the field.

47. Cross-Wires in the Solar.—The Saegmuller solar telescope is provided with three horizontal and three vertical cross-wires, as shown in Fig. 17. The spacing of the extreme wires is purposely made very nearly equal to the apparent diameter of the sun, and it is therefore very easy to equalize the margin on all sides and thus sight the telescope directly at the center of the sun. The complication of allowing for the semi-diameter of the sun, as in previous methods, is thus avoided.

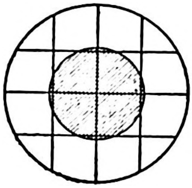


FIG. 17

THIRD METHOD OF DETERMINING AZIMUTH: BY OBSERVING POLARIS AT ELONGATION

48. General Considerations.—The most accurate method for determining azimuth is by observing Polaris at elongation. Since at that time the star is moving in a direction nearly parallel to the plane of the meridian and hence perpendicular to the plane of the horizon, its azimuth remains practically unchanged for several minutes. A method of determining azimuth by observing Polaris at elongation was given in *Transit Surveying*, Part 2. The one explained below is more accurate, although not so convenient.

49. General Formulas.—The following formulas are obtained from spherical trigonometry:

Let a = azimuth of Polaris;
δ = declination of Polaris;
t = hour angle of Polaris at elongation;
φ = observer's latitude.

Then,
$$\cos t = \frac{\tan \varphi}{\tan \delta} \qquad (1)$$
$$\sin a = \frac{\cos \delta}{\cos \varphi} \qquad (2)$$

If the right ascension of Polaris is denoted by α, the sidereal time of elongation is (see *Practical Astronomy*, Part 1) $\alpha \pm t$, the plus sign applying to western elongation, and the minus sign to eastern elongation. By reducing this sidereal time to mean time, the mean time of elongation is obtained. By measuring at that time the angle between the star and the mark whose azimuth is required, the latter azimuth is readily determined by adding the azimuth of Polaris to or subtracting it from the angle between the star and the mark.

The quantities δ and α are taken from the Ephemeris.

50. Making the Observations.—The manner in which observations are made is different from that employed in any of the preceding methods. In every other case, the vernier of the horizontal circle was first set at zero, but in this method, which is susceptible of a high degree of accuracy, it is better to distribute the readings around the horizontal circle. The telescope being directed to the reference mark, the horizontal circle is read; then, without loosening the lower clamp, the telescope is pointed to the star, and the circle is read again. The position of the lower plate is next changed, the vernier of the horizontal circle turned 180°, the telescope inverted, and the observation repeated, taking one reading on the mark and one on the star. The process is continued in the same manner, inverting the telescope and changing the position of the vernier on the horizontal plate after each complete observation. The azimuth computed by formula 2, Art. 49, is added to the angle between the star and the mark, if Polaris is at eastern elongation; otherwise, it is subtracted. The result is the azimuth of the mark.

It is not necessary to record the time, but the observations must not be extended more than 10 or 12 minutes on each side of elongation.

FOURTH METHOD OF DETERMINING AZIMUTH: BY OBSERVING A SOUTH STAR AT ANY HOUR ANGLE

51. This method requires not even the roughest knowledge of the time; the observations may be made at any time, provided only that the star observed, which may be any south star that can be identified by the observer, is at least 2 hours from the meridian; and in order to secure increased accuracy, the observations may be continued as long as desired. It is probable that this method will be preferred by the student, but he should bear in mind that when azimuth is determined by observations on Polaris at elongation (third method), the results will be more accurate than can be obtained in any other way. In the preceding method, only the vertical wire was used; in the present method both vertical and horizontal wires must be in accurate adjustment, and the star brought exactly to their intersection.

52. Method of Observing.—The instrument having been carefully leveled, clamp the lower motion, sight at the mark, and read the horizontal circle. Unclamp the upper motion, bring the intersection of the cross-wires accurately on the star, and read both the horizontal and the vertical circle. This completes one observation.

Now unclamp both lower plates, change the position of the vernier on the horizontal circle by 20° or 30°, revolve the instrument in azimuth 180°, invert the telescope, and repeat the observation. If the transit has not a complete vertical circle, it will not be possible to invert it. In this case, if the highest accuracy is desired, a mercury horizon should be used and each alternate pointing be made at the star reflected in the mercury.

Six observations, at least, should be taken as above described, the position of the lower plate being changed after each observation, and the telescope used alternately direct and inverted, if possible.

53. Rule for Computing the Azimuth.—*Subtract each reading of the horizontal circle when pointed on the mark*

from the corresponding reading when pointed on the star, and form the mean of the resulting differences. Next, form the mean of the vertical circle readings and correct this for index error if the telescope was not inverted and if a mercury horizon was not used; otherwise, no correction for index error need be applied. Correct also for refraction: the result is the true altitude of the star. Subtract the true altitude from 90°, to obtain the zenith distance.

Take from the Ephemeris, or from Table VI, Practical Astronomy, Part 1, the value of the declination. Find the azimuth of the star by applying the formula of Art. 38. Subtract the corrected mean reading of the horizontal circle from the azimuth of the star, adding 360° to the latter azimuth, if necessary. The result is the desired azimuth of the line.

It should be constantly borne in mind that in turning the instrument from the mark to the star, it should be turned toward the right.

EXAMPLE.—On January 5, 1903, at Philadelphia, the following readings were taken on the star Sirius when the star was east of the meridian. The latitude of Philadelphia being 39° 58′, find the azimuth of the mark.

Telescope	Vertical Circle			Horizontal Circle: Pointing on Mark			Horizontal Circle: Pointing on Star		
Direct . . .	20°	17′	10″	300°	27′	10″	330°	29′	0″
Inverted . .	200	17	20	114	30	0	144	33	20
Inverted . .	200	18	10	175	35	10	205	38	50
Direct . . .	20	18	30	60	24	30	90	29	30
Direct . . .	20	19	20	108	10	0	138	15	50
Inverted . .	200	20	30	3	6	10	33	12	30

SOLUTION.—To find the true altitude:

Mean of vertical circle readings	20° 18′ 30″
Refraction (Table VII, *Practical Astronomy*, Part 1)	−2 32
True altitude	20° 15′ 58″
Zenith distance = 90° − true altitude	69° 44′ 2″

To find the azimuth of the star:

$z = 69° 44' 2''$, $\varphi = 39° 58'$, $\delta = -16° 35' 9''$ (Table VI, *Practical Astronomy*, Part 1); $\frac{1}{2}(z + \varphi + \delta) = 46° 33' 27''$; $\frac{1}{2}(z + \varphi - \delta)$

= 63° 8′ 36″. Substituting these values in the formula of Art. 38, we have,

$$\sin \tfrac{1}{2} a = \sqrt{\frac{\cos 46° 33' 27'' \sin 63° 8' 36''}{\sin 69° 44' 2'' \cos 39° 58'}}$$

$$\tfrac{1}{2} a = 67° 28' 30''; \; a = 134° 57'$$

Since the star is east of the meridian, the acute angle corresponding to $\sin \frac{1}{2} a$ is used. The mean of the differences of the circle readings is 30° 4′ 20″. Subtracting this from the azimuth of the star, the azimuth of the mark is found to be 104° 52′ 40″. Ans.

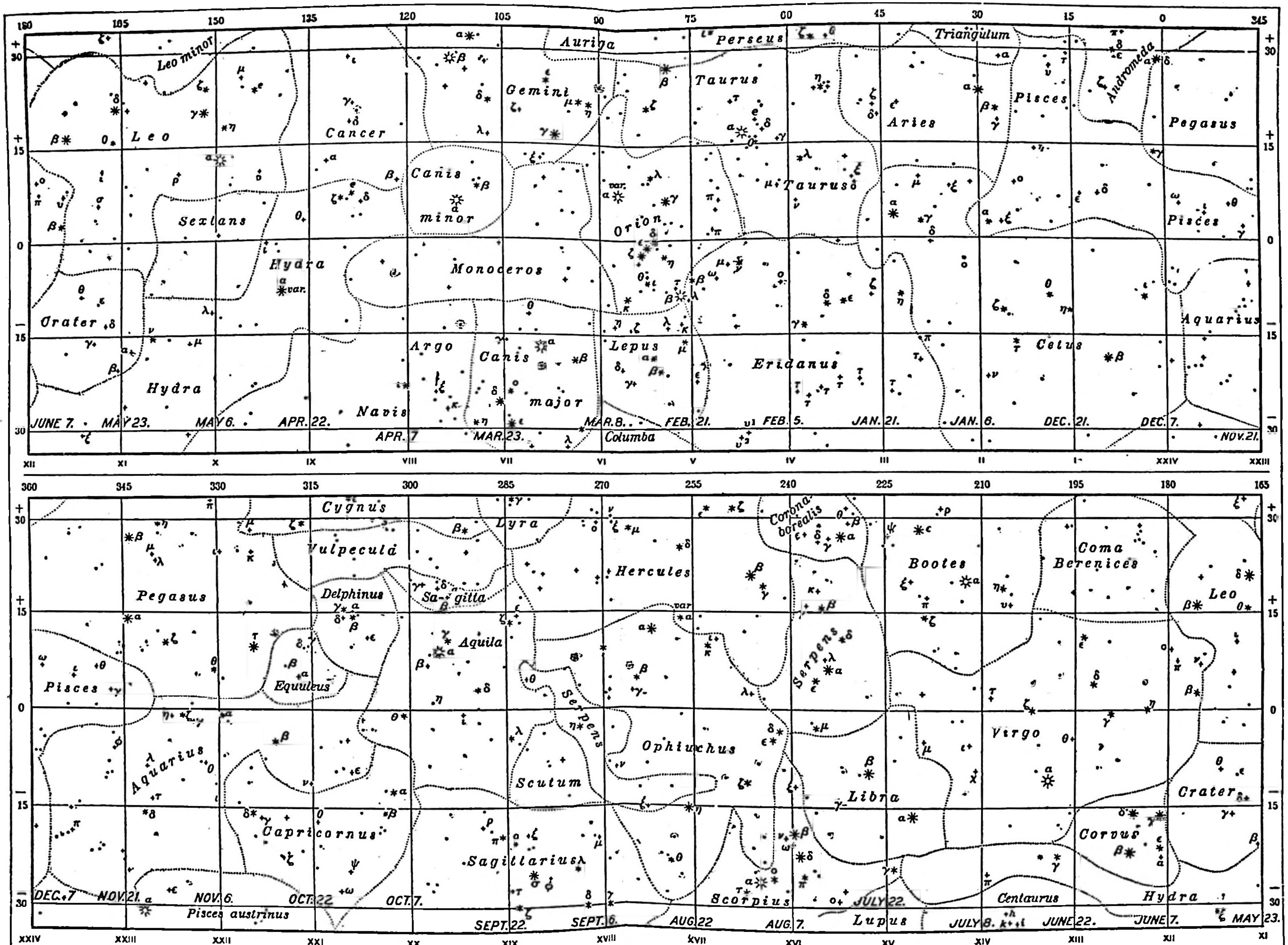
FIFTH METHOD OF DETERMINING AZIMUTH: BY OBSERVING POLARIS AT ANY HOUR ANGLE

54. Advantages of the Method.—The greatest advantage of the method about to be described is that the observations can be made at about the time of sunset. It is therefore unnecessary for the observer to place a light at the station that he is observing, or to illuminate the wires of his instrument, and this makes the method more convenient than any other in which a star is employed.

This method is more accurate than any of those in which the sun is observed, because the motion of Polaris is so slow that the error of the watch need only be known within a minute or two. This error can be quickly and easily found in the field, as described in Art. 55.

It is strongly recommended that the student use either this method or that of observing Polaris at elongation, since these are far more accurate than observations on the sun can ever be. If, however, he cannot choose his time for observing, but is compelled to make his observations at various times during the day, he should employ the methods of Arts. 34 to 41.

55. To Find the Sidereal Time and the Watch Error.—If the mean local time is known, it may be easily converted into sidereal time, as was explained in *Practical Astronomy*, Part 1. Otherwise, the sidereal time may be determined with sufficient accuracy as follows: Point the telescope at Polaris and carefully bisect the star with the vertical wire. Then revolve the telescope on the horizontal axis



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Map of Stars between 30 North and 30 South Declination.

FIG. 18

until it points toward the south. Since Polaris is always nearly in the meridian, the optical axis of the telescope, after being revolved, will point nearly toward the south.

Now point the telescope at such an elevation that one of the bright southern stars shown in the map of Fig. 18 will pass across the field. By sighting along the top of the tube, several such stars can always be found. These are, of course, all moving across the sky from east to west.

When the star chosen is seen to enter the field, clamp the vertical motion and bring the star near the horizontal wire. At the instant that it passes the vertical wire, note the watch time. Since the star is very nearly on the meridian, the true sidereal time at this instant is very nearly equal to the right ascension of the star; and hence the watch error is obtained by subtracting the time shown by the watch from the right ascension. This error, being added algebraically to any watch time, gives the corresponding sidereal time.

56. Explanation of the Map.—The map, Fig. 18, shows all the brighter south stars whose declinations lie between $-30°$ and $+30°$. The student should notice that the lower edge of the map is not parallel to the horizon, but is parallel to the celestial equator. The vertical lines are hour circles; the right ascension of each of these hour circles is written at the top of the map in degrees, and at the bottom in hours. At the bottom of the map is also written the time of the year when each of these hour circles is due south at 7 P. M. For example, on July 8, the circle whose right ascension is 14 hours is due south at 7 P. M.; that is, it then coincides with the meridian. The bright star α Bootis is then 10 minutes east of the meridian, and α Virginis is about 45 minutes west of the meridian. Either of these stars would be a good one to observe on this date.

The name of each star is found as follows: The dotted lines show the limits of the various constellations. Each star of a constellation is named by merely prefixing the Greek letter shown on the map to the name of the constellation. Thus, δ Virginis is the star δ of the constellation

Virgo. As the names of the constellations are in Latin, they are written in the genitive case when preceded by the letter of a star. Thus, α Virginis may be considered equivalent to α of Virgo.

EXAMPLE.—What star on the map has a right ascension of $11^h\ 44^m$, and a declination of $+15°\ 7'$?

SOLUTION.—Looking at the position indicated, we find a star marked β in the constellation Leo. Hence, the star is β Leonis. Ans.

EXAMPLE FOR PRACTICE

Tell the names of the stars in the following positions: (*a*) Right ascension $19^h\ 46^m$, declination $+8°\ 34'$; (*b*) right ascension $0^h\ 3^m$, declination $+28°\ 33'$; (*c*) right ascension $7^h\ 34^m$, declination $+5°\ 29'$. (*d*) What stars are well suited for observation soon after sunset on October 7?

Ans.
(*a*) α Aquilæ
(*b*) α Andromedæ
(*c*) α Canis minoris
(*d*) β and ϵ Cygni, β and γ Delphini, α, γ, δ, and θ Aquilæ, γ Sagittarii, and α and β Capricorni

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57. To Compute the Azimuth of Polaris, the Sidereal Time Being Known.

Let t = hour angle of Polaris;
α = its right ascension;
δ = its declination;
θ = sidereal time;
φ = latitude of observer;
K = quantity given in Table V;
a = azimuth of Polaris at time θ.

Then, $t = \theta - \alpha$, and a is approximately given by the formula

$$a = -\frac{\sin t}{\cos \varphi}[(90° - \delta) + K \tan \varphi]$$

EXAMPLE.—The right ascension of Polaris being $1^h\ 24^m\ 0^s$, its declination $+88°\ 47'\ 26''$, the sidereal time $9^h\ 27^m$, and the latitude of the observer $+39°\ 58'$, find the azimuth of Polaris.

SOLUTION.—In order to apply the formula, $\varphi = +39°\ 58'$, $\delta = +88°\ 47'\ 26''$, and $t = 9^h\ 27^m\ 0^s - 1^h\ 24^m\ 0^s = 8^h\ 3^m\ 0^s$, which is equal to

120° 45′. The value of K corresponding to 120.8° is found, from Table V, to be −46″. Substituting these values in the formula,

$$a = -\frac{\sin 120^\circ 45'}{\cos 39^\circ 58'}[(90^\circ - 88^\circ 47' 26'') - 46'' \times \tan 39^\circ 58'] = -1^\circ 20' 38''$$

Therefore, the azimuth of Polaris reckoned from the north point is −1° 20′ 38″. Ans.

EXAMPLE FOR PRACTICE

In the above example, find the azimuth of Polaris when the sidereal time is $9^h\ 59^m\ 30^s$. Ans. −1° 12′ 54″

NOTE.—By comparing this answer with that of the preceding example, it may be seen how slowly Polaris moves.

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58. Method of Making the Observations.—Set the transit up over the instrument station and level very carefully. Clamp the lower motion and point on the reference mark *M*. Clamp the upper motion, and, using the tangent screw, carefully bisect the mark with the vertical wire. Read and record both verniers.

Unclamp the upper motion, point on Polaris, and carefully bisect the star with the vertical wire, using the slow-motion screw as before. At the instant the bisection is perfected, read and record the exact watch time in hours, minutes, and seconds, and then read and record both verniers.

Now unclamp the upper motion, again direct the telescope to the mark, and read both verniers. Then sight again to the star, and note both the reading of the verniers and the time. Proceed in the same manner until four or five readings have been obtained. The difference between the mean of the readings on the mark and that of the readings on the star gives the difference between the azimuth of the mark and that of the star. Notice that the lower motion remains securely clamped throughout all these observations.

To make the observation that gives the watch error, point again on Polaris and clamp both motions. Then, being careful not to disturb either the upper or the lower motion, plunge the telescope and point it at such an elevation that any bright south star will pass across the field. Find the name of the star from the map (Art. 56); without disturbing the instrument in any way, look through the telescope, and

at the instant the star crosses the vertical wire, record the watch time. This completes the observation.

59. Rule for Computing the Azimuth of the Line. *Find from the Ephemeris the right ascension and declination of Polaris, and also the right ascension of the south star observed. Compute the watch error (Art. 24). Form the average of the four vernier readings on the mark and also of the vernier readings on the star, and subtract the average of the readings on Polaris from the average of the readings on the mark. The remainder is the angle between the star and the mark. Form the average of the two watch times, and add to this the watch error. The sum is the sidereal time of the observation. Compute the azimuth of Polaris (Art. 58). Add, algebraically, the azimuth of Polaris to the angle between the star and the mark. The sum will be the desired azimuth of the mark.*

EXAMPLE.—Observations made as above described gave the following results: Average of readings on the station, 108° 17′ 30″; average of readings on the star, 30° 3′ 5″; mean of recorded times, $6^h\ 25^m\ 30^s$. The telescope was pointed on α Leonis (see Fig. 18), and the watch time of its transit across the vertical thread was taken; this time was $7^h\ 1^m\ 30^s$. The latitude being +39° 58′; the right ascension of Polaris, $1^h\ 24^m\ 0^s$; its declination, +88° 47′ 26″; and the right ascension of α Leonis, $10^h\ 3^m\ 0^s$, to find the azimuth of the line.

SOLUTION.—First compute the watch error and the sidereal time.

The sidereal time of the transit of α Leonis is equal to its right ascension	$10^h\ 3^m\ 0^s$
The corresponding watch time was	7 1 30
Hence, the watch error was	$3^h\ 1^m\ 30^s$
The average watch time of the pointings on Polaris was	6 25 30
Hence, the sidereal time of the observation was .	$9^h\ 27^m\ 0^s$

Next, compute the azimuth of Polaris. (This computation is given in the example, Art. 58.) Azimuth of Polaris = −1° 20′ 38″. Finally, compute the azimuth of the line.

Reading on Station *B*	108° 17′ 30″
Reading on Polaris	30 3 5
Difference	78° 14′ 25″
Azimuth of Polaris	−1 20 38
Azimuth of line	76° 53′ 47″

Ans.

EXAMPLE FOR PRACTICE

Find the azimuth of the line observed in the following series of observations, the latitude of the observer being $+29° 26'$:

Observations in the field	Average of four readings on station . .	300° 17′ 10″
	Average of four readings on Polaris . .	306° 4′ 0″
	Average of the two watch times	7^h 3^m 0^s
	Watch time of transit of β Aquarii	7^h 22^m 20^s
Quantities found from the Ephemeris	Right ascension of Polaris	1^h 4^m 4.7^s
	Declination of Polaris	+88° 29′ 58″
	Right ascension of β Aquarii . . .	21^h 26^m 20^s

Ans. 355° 42′ 26″

TABLE I

VARIATION OF ALTITUDE IN 1^m FROM NOON

Latitude Degrees	Declination of Different Name From the Latitude											
	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°
	"	"	"	"	"	"	"	"	"	"	"	"
0					28.1	22.4	18.7	16.0	14.0	12.4	11.1	10.1
1				28.1	22.4	18.7	16.0	14.0	12.4	11.2	10.1	9.3
2			28.1	22.4	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6
3		28.1	22.4	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6	8.0
4	28.1	22.4	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6	8.0	7.4
5	22.4	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6	8.0	7.4	7.0
6	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6	8.0	7.5	7.0	6.6
7	16.0	14.0	12.4	11.2	10.2	9.3	8.6	8.0	7.5	7.0	6.6	6.2
8	14.0	12.4	11.2	10.2	9.3	8.6	8.0	7.5	7.0	6.6	6.2	5.9
9	12.4	11.2	10.2	9.3	8.6	8.0	7.5	7.0	6.6	6.2	5.9	5.6
10	11.1	10.1	9.3	8.6	8.0	7.4	7.0	6.6	6.2	5.9	5.6	5.3
11	10.1	9.3	8.6	8.0	7.4	7.0	6.6	6.2	5.9	5.6	5.3	5.1
12	9.2	8.5	7.9	7.4	7.0	6.5	6.2	5.9	5.6	5.3	5.0	4.8
13	8.5	7.9	7.4	6.9	6.5	6.2	5.8	5.6	5.3	5.0	4.8	4.6
14	7.9	7.4	6.9	6.5	6.2	5.8	5.5	5.3	5.0	4.8	4.6	4.4
15	7.3	6.9	6.5	6.1	5.8	5.5	5.3	5.0	4.8	4.6	4.4	4.2
16	6.8	6.5	6.1	5.8	5.5	5.2	5.0	4.8	4.6	4.4	4.2	4.1
17	6.4	6.1	5.8	5.5	5.2	5.0	4.8	4.6	4.4	4.2	4.1	3.9
18	6.0	5.7	5.5	5.2	5.0	4.8	4.6	4.4	4.2	4.1	3.9	3.8
19	5.7	5.4	5.2	4.9	4.7	4.5	4.4	4.2	4.0	3.9	3.8	3.6
20	5.4	5.1	4.9	4.7	4.5	4.3	4.2	4.0	3.9	3.8	3.6	3.5
21	5.1	4.9	4.7	4.5	4.3	4.2	4.0	3.9	3.7	3.6	3.5	3.4
22	4.9	4.7	4.5	4.3	4.1	4.0	3.9	3.7	3.6	3.5	3.4	3.3
23	4.6	4.4	4.3	4.1	4.0	3.8	3.7	3.6	3.5	3.4	3.3	3.2
24	4.4	4.2	4.1	3.9	3.8	3.7	3.6	3.5	3.4	3.3	3.2	3.1
25	4.2	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.1	3.0
26	4.0	3.9	3.8	3.6	3.5	3.4	3.3	3.2	3.1	3.0	3.0	2.9
27	3.9	3.7	3.6	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.9	2.8
28	3.7	3.6	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	2.8	2.7
29	3.5	3.4	3.3	3.2	3.1	3.1	3.0	2.9	2.8	2.8	2.7	2.6
30	3.4	3.3	3.2	3.1	3.0	3.0	2.9	2.8	2.7	2.7	2.6	2.5
31	3.3	3.2	3.1	3.0	2.9	2.9	2.8	2.7	2.6	2.6	2.5	2.5
32	3.1	3.1	3.0	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.5	2.4
33	3.0	2.9	2.9	2.8	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3
34	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.5	2.4	2.4	2.3	2.3
35	2.8	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2
36	2.7	2.6	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.1
37	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.2	2.1	2.1
38	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.1	2.0
39	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.1	2.0	2.0	2.0
40	2.3	2.3	2.2	2.2	2.2	2.1	2.1	2.0	2.0	2.0	1.9	1.9
41	2.3	2.2	2.2	2.1	2.1	2.1	2.0	2.0	1.9	1.9	1.9	1.8
42	2.2	2.1	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8
43	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7
44	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7
45	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.7	1.6
46	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6
47	1.8	1.8	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.6
48	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.6	1.5	1.5
49	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.5
50	1.6	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.4	1.4
52	1.5	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.4	1.3
54	1.4	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.3
56	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.2
58	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1	1.1
60	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0
62	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.9
64	1.0	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
66	0.9	0.9	0.9	0.9	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
68	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7
70	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7

TABLE I—*Continued*

Latitude Degrees	Declination of Different Name From the Latitude												
	12°	13°	14°	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°
	"	"	"	"	"	"	"	"	"	"	"	"	"
0	9.2	8.5	7.9	7.3	6.8	6.4	6.0	5.7	5.4	5.1	4.9	4.6	4.4
1	8.5	7.9	7.4	6.9	6.5	6.1	5.7	5.4	5.1	4.9	4.7	4.4	4.2
2	7.9	7.4	6.9	6.5	6.1	5.8	5.5	5.2	4.9	4.7	4.5	4.3	4.1
3	7.4	6.9	6.5	6.1	5.8	5.5	5.2	4.9	4.7	4.5	4.3	4.1	3.9
4	7.0	6.5	6.2	5.8	5.5	5.2	5.0	4.7	4.5	4.3	4.1	4.0	3.8
5	6.5	6.2	5.8	5.5	5.2	5.0	4.8	4.5	4.3	4.2	4.0	3.8	3.7
6	6.2	5.8	5.5	5.3	5.0	4.8	4.6	4.4	4.2	4.0	3.9	3.7	3.6
7	5.9	5.6	5.3	5.0	4.8	4.6	4.4	4.2	4.0	3.9	3.7	3.6	3.5
8	5.6	5.3	5.0	4.8	4.6	4.4	4.2	4.0	3.9	3.7	3.6	3.5	3.4
9	5.3	5.0	4.8	4.6	4.4	4.2	4.1	3.9	3.8	3.6	3.5	3.4	3.3
10	5.0	4.8	4.6	4.4	4.2	4.1	3.9	3.8	3.6	3.5	3.4	3.3	3.2
11	4.8	4.6	4.4	4.2	4.1	3.9	3.8	3.6	3.5	3.4	3.3	3.2	3.1
12	4.6	4.4	4.3	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0
13	4.4	4.3	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9
14	4.2	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8
15	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	2.8
16	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	2.8	2.7
17	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	2.8	2.7	2.6
18	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.9	2.8	2.7	2.6	2.5
19	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.9	2.8	2.7	2.6	2.6	2.5
20	3.4	3.3	3.2	3.1	3.0	2.9	2.9	2.8	2.7	2.6	2.6	2.5	2.4
21	3.3	3.2	3.1	3.0	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.4	2.4
22	3.2	3.1	3.0	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.4	2.4	2.3
23	3.1	3.0	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.4	2.4	2.3	2.3
24	3.0	2.9	2.8	2.8	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2
25	2.9	2.8	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2
26	2.8	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.1	2.1
27	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.2	2.2	2.1	2.1	2.1
28	2.6	2.6	2.5	2.5	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.1	2.0
29	2.6	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.0	2.0	2.0
30	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.0	2.0	2.0	1.9
31	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.0	2.0	2.0	1.9	1.9
32	2.3	2.3	2.2	2.2	2.2	2.1	2.1	2.0	2.0	1.9	1.9	1.9	1.8
33	2.3	2.2	2.2	2.1	2.1	2.1	2.0	2.0	1.9	1.9	1.9	1.8	1.8
34	2.2	2.2	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8
35	2.2	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.8	1.8	1.8	1.7	1.7
36	2.1	2.1	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7
37	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.6
38	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.8	1.7	1.7	1.7	1.6	1.6
39	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.6	1.6	1.6	1.6
40	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5
41	1.8	1.8	1.8	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5
42	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5
43	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.4	1.4
44	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.4	1.4	1.4
45	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4
46	1.6	1.6	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.3	1.3
47	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.3
48	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3
49	1.4	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2
50	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2
52	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.1	1.1
54	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.1
56	1.2	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0
58	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
60	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.9	0.9	0.9	0.9	0.9	0.9
62	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8
64	0.9	0.9	0.9	0.9	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
66	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	
68	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7			
70	0.7	0.7	0.7	0.6	0.6	0.6	0.6	0.6					

TABLE I—*Continued*

Latitude Degrees	Declination of Same Name as the Latitude											
	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°
	″	″	″	″	″	″	″	″	″	″	″	″
0					28.1	22.4	18.7	16.0	14.0	12.4	11.1	10.1
1						28.0	22.4	18.6	16.0	13.9	12.4	11.1
2							28.0	22.3	18.6	15.9	13.9	12.3
3								27.9	22.3	18.5	15.8	13.8
4	28.1								27.8	22.2	18.5	15.8
5	22.4	28.0								27.7	22.1	18.4
6	18.7	22.4	28.0								27.6	22.0
7	16.0	18.6	22.3	27.9								27.4
8	14.0	16.0	18.6	22.3	27.8							
9	12.4	13.9	15.9	18.5	22.2	27.7						
10	11.1	12.4	13.9	15.8	18.5	22.1	27.6					
11	10.1	11.1	12.3	13.8	15.8	18.4	22.0	27.4				
12	9.2	10.1	11.1	12.3	13.8	15.7	18.3	21.9	27.3			
13	8.5	9.2	10.0	11.0	12.2	13.7	15.6	18.2	21.7	27.1		
14	7.9	8.5	9.2	10.0	10.9	12.1	13.6	15.5	18.0	21.6	26.9	
15	7.3	7.8	8.4	9.1	9.9	10.9	12.1	13.5	15.4	17.9	21.4	26.7
16	6.8	7.3	7.8	8.4	9.1	9.8	10.8	12.0	13.4	15.3	17.8	21.3
17	6.4	6.8	7.2	7.8	8.3	9.0	9.8	10.7	11.9	13.3	15.2	17.6
18	6.0	6.4	6.8	7.2	7.7	8.3	8.9	9.7	10.6	11.8	13.2	15.0
19	5.7	6.0	6.3	6.7	7.2	7.6	8.2	8.9	9.6	10.6	11.7	13.1
20	5.4	5.7	6.0	6.3	6.7	7.1	7.6	8.1	8.8	9.5	10.5	11.6
21	5.1	5.4	5.6	5.9	6.3	6.6	7.0	7.5	8.1	8.7	9.5	10.4
22	4.9	5.1	5.3	5.6	5.9	6.2	6.6	7.0	7.5	8.0	8.6	9.4
23	4.6	4.8	5.0	5.3	5.5	5.8	6.1	6.5	6.9	7.4	7.9	8.5
24	4.4	4.6	4.8	5.0	5.2	5.5	5.8	6.1	6.4	6.8	7.3	7.8
25	4.2	4.4	4.6	4.7	5.0	5.2	5.4	5.7	6.0	6.4	6.8	7.2
26	4.0	4.2	4.3	4.5	4.7	4.9	5.1	5.4	5.7	6.0	6.3	6.7
27	3.9	4.0	4.1	4.3	4.5	4.7	4.9	5.1	5.3	5.6	5.9	6.2
28	3.7	3.8	4.0	4.1	4.3	4.4	4.6	4.8	5.0	5.3	5.5	5.8
29	3.5	3.7	3.8	3.9	4.1	4.2	4.4	4.6	4.7	5.0	5.2	5.5
30	3.4	3.5	3.6	3.7	3.9	4.0	4.2	4.3	4.5	4.7	4.9	5.1
31	3.3	3.4	3.5	3.6	3.7	3.8	4.0	4.1	4.3	4.4	4.6	4.8
32	3.1	3.2	3.3	3.4	3.5	3.7	3.8	3.9	4.1	4.2	4.4	4.6
33	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.9	4.0	4.2	4.3
34	2.9	3.0	3.1	3.2	3.2	3.3	3.4	3.6	3.7	3.8	3.9	4.1
35	2.8	2.9	3.0	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.9
36	2.7	2.8	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7
37	2.6	2.7	2.7	2.8	2.9	2.9	3.0	3.1	3.2	3.3	3.4	3.5
38	2.5	2.6	2.6	2.7	2.8	2.8	2.9	3.0	3.0	3.2	3.2	3.3
39	2.4	2.5	2.5	2.6	2.7	2.7	2.8	2.9	2.9	3.0	3.1	3.2
40	2.3	2.4	2.4	2.5	2.6	2.6	2.7	2.7	2.8	2.9	3.0	3.0
41	2.3	2.3	2.4	2.4	2.5	2.5	2.6	2.6	2.7	2.8	2.8	2.9
42	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.5	2.6	2.6	2.7	2.8
43	2.1	2.1	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.5	2.6	2.7
44	2.0	2.1	2.1	2.1	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.5
45	2.0	2.0	2.0	2.1	2.1	2.2	2.2	2.2	2.3	2.3	2.4	2.4
46	1.9	1.9	2.0	2.0	2.0	2.1	2.1	2.2	2.2	2.2	2.3	2.3
47	1.8	1.9	1.9	1.9	2.0	2.0	2.0	2.1	2.1	2.1	2.2	2.2
48	1.8	1.8	1.8	1.9	1.9	1.9	2.0	2.0	2.0	2.1	2.1	2.1
49	1.7	1.7	1.8	1.8	1.8	1.8	1.9	1.9	1.9	2.0	2.0	2.1
50	1.6	1.7	1.7	1.7	1.8	1.8	1.8	1.8	1.9	1.9	1.9	2.0
52	1.5	1.6	1.6	1.6	1.6	1.6	1.7	1.7	1.7	1.8	1.8	1.8
54	1.4	1.4	1.5	1.5	1.5	1.5	1.5	1.6	1.6	1.6	1.6	1.7
56	1.3	1.3	1.4	1.4	1.4	1.4	1.4	1.4	1.5	1.5	1.5	1.5
58	1.2	1.2	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.4	1.4	1.4
60	1.1	1.1	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.3	1.3
62	1.0	1.0	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.2	1.2
64	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.1
66	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	1.0
68	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.9	0.9
70	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.8	0.8	0.8	0.8

TABLE I—*Continued*

Latitude Degrees	Declination of Same Name as the Latitude												
	12°	13°	14°	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°
	"	"	"	"	"	"	"	"	"	"	"	"	"
0	9.2	8.5	7.9	7.3	6.8	6.4	6.0	5.7	5.4	5.1	4.9	4.6	4.4
1	10.1	9.2	8.5	7.8	7.3	6.8	6.4	6.0	5.7	5.4	5.1	4.8	4.6
2	11.1	10.0	9.2	8.4	7.8	7.2	6.8	6.3	6.0	5.6	5.3	5.0	4.8
3	12.3	11.0	10.0	9.1	8.4	7.8	7.2	6.7	6.3	5.9	5.6	5.3	5.0
4	13.8	12.2	10.9	9.9	9.1	8.3	7.7	7.2	6.7	6.3	5.9	5.5	5.2
5	15.7	13.7	12.1	10.9	9.8	9.0	8.3	7.6	7.1	6.6	6.2	5.8	5.5
6	18.3	15.6	13.6	12.1	10.8	9.8	8.9	8.2	7.6	7.0	6.6	6.1	5.8
7	21.9	18.2	15.5	13.5	12.0	10.7	9.7	8.9	8.1	7.5	7.0	6.5	6.1
8	27.3	21.7	18.0	15.4	13.4	11.9	10.6	9.6	8.8	8.1	7.5	6.9	6.4
9		27.1	21.6	17.9	15.3	13.3	11.8	10.6	9.5	8.7	8.0	7.4	6.8
10			26.9	21.4	17.8	15.2	13.2	11.7	10.5	9.5	8.6	7.9	7.3
11				26.7	21.3	17.6	15.0	13.1	11.6	10.4	9.4	8.5	7.8
12					26.5	21.1	17.5	14.9	13.0	11.5	10.3	9.3	8.4
13						26.2	20.9	17.3	14.8	12.8	11.3	10.1	9.2
14							26.0	20.7	17.1	14.6	12.7	11.2	10.0
15								25.7	20.4	16.9	14.4	12.5	11.1
16	26.5								25.4	20.2	16.7	14.3	12.4
17	21.1	26.2								25.1	20.0	16.5	14.1
18	17.5	20.9	26.0								24.8	19.7	16.3
19	14.9	17.3	20.7	25.7								24.5	19.5
20	13.0	14.8	17.1	20.4	25.4								24.2
21	11.5	12.8	14.6	16.9	20.2	25.1							
22	10.3	11.3	12.7	14.4	16.7	20.0	24.8						
23	9.3	10.1	11.2	12.5	14.3	16.5	19.7	24.5					
24	8.4	9.2	10.0	11.1	12.4	14.1	16.3	19.5	24.2				
25	7.7	8.3	9.0	9.9	10.9	12.2	13.9	16.1	19.2	23.8			
26	7.1	7.6	8.2	8.9	9.8	10.8	12.1	13.7	15.9	18.9	23.5		
27	6.6	7.0	7.5	8.1	8.8	9.6	10.6	11.9	13.5	15.6	18.6	23.1	
28	6.2	6.5	7.0	7.4	8.0	8.7	9.5	10.5	11.7	13.3	15.4	18.3	22.7
29	5.7	6.1	6.4	6.9	7.3	7.9	8.6	9.4	10.3	11.5	13.1	15.1	18.0
30	5.4	5.7	6.0	6.4	6.8	7.2	7.8	8.4	9.2	10.1	11.3	12.8	14.9
31	5.1	5.3	5.6	5.9	6.3	6.7	7.1	7.7	8.3	9.0	10.0	11.1	12.6
32	4.8	5.0	5.2	5.5	5.8	6.2	6.5	7.0	7.5	8.1	8.9	9.8	10.9
33	4.5	4.7	4.9	5.1	5.4	5.7	6.1	6.4	6.9	7.4	8.0	8.7	9.6
34	4.3	4.4	4.6	4.8	5.1	5.3	5.6	5.9	6.3	6.8	7.3	7.8	8.6
35	4.0	4.2	4.4	4.5	4.7	5.0	5.2	5.5	5.8	6.2	6.6	7.1	7.7
36	3.8	4.0	4.1	4.3	4.5	4.7	4.9	5.1	5.4	5.7	6.1	6.5	7.0
37	3.6	3.8	3.9	4.0	4.2	4.4	4.6	4.8	5.0	5.3	5.6	6.0	6.4
38	3.4	3.6	3.7	3.8	4.0	4.1	4.3	4.5	4.7	4.9	5.2	5.5	5.8
39	3.3	3.4	3.5	3.6	3.8	3.9	4.0	4.2	4.4	4.6	4.8	5.1	5.4
40	3.1	3.2	3.3	3.4	3.6	3.7	3.8	4.0	4.1	4.3	4.5	4.7	5.0
41	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.9	4.0	4.2	4.4	4.6
42	2.9	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.7	3.8	4.0	4.1	4.3
43	2.7	2.8	2.9	3.0	3.0	3.1	3.2	3.3	3.5	3.6	3.7	3.9	4.0
44	2.6	2.7	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.8
45	2.5	2.6	2.6	2.7	2.8	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5
46	2.4	2.4	2.5	2.6	2.6	2.7	2.8	2.8	2.9	3.0	3.1	3.2	3.3
47	2.3	2.3	2.4	2.4	2.5	2.6	2.6	2.7	2.8	2.9	2.9	3.0	3.1
48	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.6	2.7	2.8	2.9	3.0
49	2.1	2.1	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.6	2.7	2.8
50	2.0	2.0	2.1	2.1	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.6
52	1.8	1.9	1.9	1.9	2.0	2.0	2.1	2.1	2.1	2.2	2.2	2.3	2.4
54	1.7	1.7	1.7	1.8	1.8	1.8	1.9	1.9	1.9	2.0	2.0	2.1	2.1
56	1.5	1.6	1.6	1.6	1.6	1.7	1.7	1.7	1.8	1.8	1.8	1.9	1.9
58	1.4	1.4	1.5	1.5	1.5	1.5	1.5	1.6	1.6	1.6	1.6	1.7	1.7
60	1.3	1.3	1.3	1.3	1.4	1.4	1.4	1.4	1.4	1.5	1.5	1.5	1.5
62	1.2	1.2	1.2	1.2	1.2	1.2	1.3	1.3	1.3	1.3	1.3	1.3	1.4
64	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.2	1.2	1.2	1.2	1.2	1.2
66	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.1	1.1	1.1	1.1
68	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	1.0	1.0
70	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.9

TABLE II

SQUARES OF INTERVAL

Seconds	Time From Meridian Passage												
	0m	1m	2m	3m	4m	5m	6m	7m	8m	9m	10m	11m	12m
0	0.0	1.0	4.0	9.0	16.0	25.0	36.0	49.0	64.0	81.0	100.0	121.0	144.0
1	0.0	1.0	4.1	9.1	16.1	25.2	36.2	49.2	64.3	81.3	100.3	121.4	144.4
2	0.0	1.1	4.1	9.2	16.3	25.3	36.4	49.5	64.5	81.6	100.7	121.7	144.8
3	0.0	1.1	4.2	9.3	16.4	25.5	36.6	49.7	64.8	81.9	101.0	122.1	145.2
4	0.0	1.1	4.3	9.4	16.5	25.7	36.8	49.9	65.1	82.2	101.3	122.5	145.6
5	0.0	1.2	4.3	9.5	16.7	25.8	37.0	50.2	65.3	82.5	101.7	122.9	146.0
6	0.0	1.2	4.4	9.6	16.8	26.0	37.2	50.4	65.6	82.8	102.0	123.2	146.4
7	0.0	1.2	4.5	9.7	16.9	26.2	37.4	50.6	65.9	83.1	102.3	123.6	146.8
8	0.0	1.3	4.6	9.8	17.1	26.4	37.6	50.9	66.1	83.4	102.7	124.0	147.2
9	0.0	1.3	4.6	9.9	17.2	26.5	37.8	51.1	66.4	83.7	103.0	124.3	147.6
10	0.0	1.4	4.7	10.0	17.4	26.7	38.0	51.4	66.7	84.0	103.4	124.7	148.0
11	0.0	1.4	4.8	10.1	17.5	26.9	38.2	51.6	67.0	84.3	103.7	125.1	148.4
12	0.0	1.4	4.8	10.2	17.6	27.0	38.4	51.8	67.2	84.6	104.0	125.4	148.8
13	0.0	1.5	4.9	10.3	17.8	27.2	38.6	52.1	67.5	84.9	104.4	125.8	149.2
14	0.1	1.5	5.0	10.5	17.9	27.4	38.9	52.3	67.8	85.3	104.7	126.2	149.7
15	0.1	1.6	5.1	10.6	18.1	27.6	39.1	52.6	68.1	85.6	105.1	126.6	150.1
16	0.1	1.6	5.1	10.7	18.2	27.7	39.3	52.8	68.3	85.9	105.4	126.9	150.5
17	0.1	1.6	5.2	10.8	18.3	27.9	39.5	53.0	68.6	86.2	105.7	127.3	150.9
18	0.1	1.7	5.3	10.9	18.5	28.1	39.7	53.3	68.8	86.5	106.1	127.7	151.3
19	0.1	1.7	5.4	11.0	18.6	28.3	39.9	53.5	69.2	86.8	106.4	128.1	151.7
20	0.1	1.8	5.4	11.1	18.8	28.4	40.1	53.8	69.4	87.1	106.8	128.4	152.1
21	0.1	1.8	5.5	11.2	18.9	28.6	40.3	54.0	69.7	87.4	107.1	128.8	152.5
22	0.1	1.9	5.6	11.3	19.1	28.8	40.5	54.3	70.0	87.7	107.5	129.2	152.9
23	0.1	1.9	5.7	11.4	19.2	29.0	40.7	54.5	70.3	88.0	107.8	129.6	153.3
24	0.2	2.0	5.8	11.6	19.4	29.2	41.0	54.8	70.6	88.4	108.2	130.0	153.8
25	0.2	2.0	5.8	11.7	19.5	29.3	41.2	55.0	70.8	88.7	108.5	130.3	154.2
26	0.2	2.1	5.9	11.8	19.7	29.5	41.4	55.3	71.1	89.0	108.9	130.7	154.6
27	0.2	2.1	6.0	11.9	19.8	29.7	41.6	55.5	71.4	89.3	109.2	131.1	155.0
28	0.2	2.2	6.1	12.0	20.0	29.9	41.8	55.8	71.7	89.6	109.6	131.5	155.4
29	0.2	2.2	6.2	12.1	20.1	30.1	42.0	56.0	72.0	89.9	109.9	131.9	155.8
30	0.2	2.2	6.2	12.2	20.2	30.2	42.2	56.2	72.2	90.2	110.2	132.2	156.2
31	0.3	2.3	6.3	12.4	20.4	30.4	42.5	56.5	72.5	90.6	110.6	132.6	156.7
32	0.3	2.4	6.4	12.5	20.6	30.6	42.7	56.8	72.8	90.9	111.0	133.0	157.1
33	0.3	2.4	6.5	12.6	20.7	30.8	42.9	57.0	73.1	91.2	111.3	133.4	157.5
34	0.3	2.5	6.6	12.7	20.9	31.0	43.1	57.3	73.4	91.5	111.7	133.8	157.9
35	0.3	2.5	6.7	12.8	21.0	31.2	43.3	57.5	73.7	91.8	112.0	134.2	158.3
36	0.4	2.6	6.8	13.0	21.2	31.4	43.6	57.8	74.0	92.2	112.4	134.6	158.8
37	0.4	2.6	6.8	13.1	21.3	31.5	43.8	58.0	74.3	92.5	112.7	134.9	159.2
38	0.4	2.7	6.9	13.2	21.5	31.7	44.0	58.3	74.5	92.8	113.1	135.3	159.6
39	0.4	2.7	7.0	13.3	21.6	31.9	44.2	58.5	74.8	93.1	113.4	135.7	160.0
40	0.4	2.8	7.1	13.4	21.8	32.1	44.4	58.8	75.1	93.4	113.8	136.1	160.4
41	0.5	2.8	7.2	13.6	21.9	32.3	44.7	59.0	75.4	93.8	114.1	136.5	160.9
42	0.5	2.9	7.3	13.7	22.1	32.5	44.9	59.3	75.7	94.1	114.5	136.9	161.3
43	0.5	2.9	7.4	13.8	22.2	32.7	45.1	59.5	76.0	94.4	114.8	137.3	161.7
44	0.5	3.0	7.5	13.9	22.4	32.9	45.3	59.8	76.3	94.7	115.2	137.7	162.1
45	0.6	3.1	7.6	14.1	22.6	33.1	45.6	60.1	76.6	95.1	115.6	138.1	162.6
46	0.6	3.1	7.7	14.2	22.7	33.3	45.8	60.3	76.9	95.4	115.9	138.5	163.0
47	0.6	3.2	7.7	14.3	22.9	33.4	46.0	60.6	77.1	95.7	116.3	138.8	163.4
48	0.6	3.2	7.8	14.4	23.0	33.6	46.2	60.8	77.4	96.0	116.6	139.2	163.8
49	0.7	3.3	7.9	14.6	23.2	33.8	46.5	61.1	77.7	96.4	117.0	139.6	164.3
50	0.7	3.4	8.0	14.7	23.4	34.0	46.7	61.4	78.0	96.7	117.4	140.0	164.7
51	0.7	3.4	8.1	14.8	23.5	34.2	46.9	61.6	78.3	97.0	117.7	140.4	165.1
52	0.8	3.5	8.2	15.0	23.7	34.4	47.2	61.9	78.6	97.4	118.1	140.8	165.6
53	0.8	3.5	8.3	15.1	23.8	34.6	47.4	62.1	78.9	97.7	118.4	141.2	166.0
54	0.8	3.6	8.4	15.2	24.0	34.8	47.6	62.4	79.2	98.0	118.8	141.6	166.4
55	0.8	3.7	8.5	15.3	24.2	35.0	47.8	62.7	79.5	98.3	119.2	142.0	166.8
56	0.9	3.7	8.6	15.5	24.3	35.2	48.1	62.9	79.8	98.7	119.5	142.4	167.3
57	0.9	3.8	8.7	15.6	24.5	35.4	48.3	63.2	80.1	99.0	119.9	142.8	167.7
58	0.9	3.9	8.8	15.7	24.7	35.6	48.5	63.5	80.4	99.3	120.3	143.2	168.1
59	1.0	3.9	8.9	15.9	24.8	35.8	48.8	63.7	80.7	99.7	120.6	143.6	168.6

TABLE III

THE SECOND CORRECTION TO BE APPLIED WHEN DETERMINING LATITUDE FROM POLARIS*

Arguments: Sidereal Time and Altitude (Additive)

Sidereal Time		Altitude 10°		20°		30°		40°		50°		60°		70°		Sidereal Time	
h	m	′	″	′	″	′	″	′	″	′	″	′	″	′	″	h	m
0	0	0	1	0	2	0	3	0	5	0	7	0	10	0	16	12	0
	30	0	0	0	1	0	1	0	2	0	3	0	4	0	6		30
1	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	13	0
	30	0	0	0	0	0	0	0	0	0	0	0	0	0	0		30
2	0	0	0	0	0	0	1	0	1	0	2	0	2	0	3	14	0
	30	0	1	0	1	0	2	0	3	0	5	0	7	0	11		30
3	0	0	1	0	3	0	5	0	7	0	10	0	14	0	22	15	0
	30	0	2	0	5	0	8	0	11	0	16	0	23	0	36		30
4	0	0	3	0	7	0	11	0	16	0	22	0	33	0	52	16	0
	30	0	4	0	9	0	14	0	21	0	30	0	43	1	8		30
5	0	0	5	0	11	0	18	0	26	0	37	0	53	1	25	17	0
	30	0	6	0	13	0	21	0	30	0	43	1	3	1	40		30
6	0	0	7	0	15	0	24	0	34	0	49	1	10	1	52	18	0
	30	0	8	0	16	0	25	0	37	0	52	1	16	2	1		30
7	0	0	8	0	17	0	27	0	39	0	55	1	20	2	7	19	0
	30	0	8	0	17	0	27	0	39	0	55	1	20	2	8		30
8	0	0	8	0	16	0	26	0	38	0	54	1	18	2	4	20	0
	30	0	7	0	15	0	25	0	36	0	51	1	14	1	57		30
9	0	0	7	0	14	0	22	0	32	0	46	1	7	1	46	21	0
	30	0	6	0	12	0	19	0	28	0	40	0	58	1	32		30
10	0	0	5	0	10	0	16	0	23	0	33	0	48	1	16	22	0
	30	0	4	0	8	0	12	0	18	0	26	0	37	0	59		30
11	0	0	3	0	6	0	9	0	13	0	19	0	27	0	43	23	0
	30	0	2	0	4	0	6	0	9	0	12	0	18	0	28		30
12	0	0	1	0	2	0	3	0	5	0	7	0	10	0	16	24	0

* For use of table, see Art. 17.

TABLE IV

EFFECT OF REFRACTION ON DECLINATION, FOR USE IN DETERMINING AZIMUTH BY THE SOLAR TRANSIT*

Latitude	Hour Angle	Declination + 23° 27′	+ 20°	+ 15°	+ 10°	+ 5°	0°	− 5°	− 10°	− 15°	− 20°	− 23° 27′
		′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
+20°	0	0 4	0 0	0 5	0 10	0 15	0 21	0 27	0 33	0 40	0 48	0 55
	1	0 3	0 0	0 6	0 11	0 16	0 22	0 28	0 34	0 41	0 49	0 56
	2	0 1	0 3	0 8	0 13	0 18	0 24	0 30	0 37	0 45	0 54	1 00
	3	0 4	0 7	0 12	0 18	0 23	0 30	0 36	0 44	0 52	1 02	1 11
	4	0 13	0 17	0 22	0 28	0 35	0 42	0 50	1 00	1 11	1 25	1 36
	5	0 34	0 39	0 47	0 57	1 07	1 19	1 36	1 57	2 28	3 14	4 01
	6	2 06	2 27	3 12	4 24	6 43	11 56					
+30°	0	0 07	0 10	0 15	0 21	0 27	0 33	0 40	0 48	0 58	1 09	1 18
	1	0 08	0 11	0 16	0 22	0 28	0 35	0 42	0 50	0 59	1 11	1 20
	2	0 10	0 14	0 19	0 25	0 32	0 39	0 46	0 55	1 06	1 18	1 27
	3	0 16	0 20	0 26	0 32	0 40	0 47	0 56	1 06	1 19	1 35	1 51
	4	0 28	0 32	0 39	0 47	0 56	1 06	1 19	1 36	1 58	2 28	2 59
	5	0 53	0 59	1 10	1 24	1 43	2 06	2 41	3 36	5 18	8 54	16 42
	6	2 11	2 33	3 23	4 51	8 00	17 27					
+40°	0	0 17	0 21	0 27	0 33	0 40	0 48	0 58	1 09	1 22	1 40	1 55
	1	0 18	0 22	0 28	0 34	0 42	0 50	1 00	1 11	1 25	1 43	2 00
	2	0 22	0 26	0 32	0 39	0 47	0 56	1 07	1 19	1 36	1 58	2 18
	3	0 29	0 33	0 41	0 48	0 58	1 09	1 22	1 39	2 02	2 36	3 09
	4	0 41	0 47	0 56	1 07	1 20	1 36	1 58	2 30	3 19	4 47	6 45
	5	1 07	1 16	1 31	1 51	2 20	3 02	4 15	6 47	14 18		
	6	2 11	2 35	3 27	5 03	8 42	22 26					
+50°	0	0 29	0 33	0 40	0 48	0 58	1 09	1 22	1 40	2 03	2 37	3 12
	1	0 30	0 34	0 42	0 50	1 00	1 11	1 25	1 44	2 09	2 46	3 24
	2	0 34	0 39	0 47	0 56	1 07	1 19	1 36	1 58	2 29	3 18	4 13
	3	0 42	0 48	0 56	1 07	1 21	1 37	1 59	2 32	3 22	4 55	7 02
	4	0 55	1 02	1 15	1 29	1 49	2 17	2 58	4 07	6 32	13 37	
	5	1 20	1 31	1 51	2 20	3 02	4 16	6 52	15 08			
	6	2 11	2 37	3 30	5 09	9 12	26 44					
+60°	0	0 43	0 48	0 58	1 09	1 22	1 40	2 03	2 37	3 32	5 16	7 46
	1	0 44	0 50	0 59	1 11	1 25	1 43	2 08	2 44	3 46	5 44	8 45
	2	0 48	0 54	1 05	1 18	1 34	1 55	2 26	3 11	4 34	7 44	13 41
	3	0 56	1 03	1 16	1 31	1 51	2 21	3 04	4 19	7 05	16 40	
	4	1 10	1 19	1 35	1 57	2 28	3 17	4 44	8 08	22 10		
	5	1 32	1 46	2 12	2 50	3 54	6 21	12 17				
	6	2 12	2 37	3 31	5 12	9 30	30 13					
Latitude	Hour Angle	+ 23° 27′	+ 20°	+ 15°	+ 10°	+ 5°	0°	− 5°	− 10°	− 15°	− 20°	− 23° 27′
		Declination										

*For use of table, see Art. 43.

TABLE V

FOR COMPUTING AZIMUTH BY POLARIS WHEN OBSERVED AT ANY HOUR ANGLE

Hour Angle Degrees	*K* Seconds	Hour Angle Degrees	Hour Angle Degrees	*K* Seconds	Hour Angle Degrees
0	+90	360	90	0	270
5	+90	355	95	− 8	265
10	+89	350	100	−16	260
15	+87	345	105	−23	255
20	+85	340	110	−31	250
25	+82	335	115	−38	245
30	+78	330	120	−45	240
35	+74	325	125	−52	235
40	+69	320	130	−58	230
45	+64	315	135	−64	225
50	+58	310	140	−69	220
55	+52	305	145	−74	215
60	+45	300	150	−78	210
65	+38	295	155	−82	205
70	+31	290	160	−85	200
75	+23	285	165	−87	195
80	+16	280	170	−89	190
85	+ 8	275	175	−90	185
90	0	270	180	−90	180

PRACTICAL ASTRONOMY

(PART 2)

EXAMINATION QUESTIONS

(1) The meridian altitude of Polaris was measured at Philadelphia on January 2, 1903, with a transit by the method of Art. 5. The following were the readings of the vertical circle:

Telescope direct	41° 11′ 25″
Telescope inverted	318 48 30
Telescope direct	41 11 55
Telescope inverted	318 48 30

Find the latitude. Ans. 39° 58′ 12.2″

(2) On January 25, 1903, the following series of altitudes were measured on the star Sirius when this star was passing the meridian. The index error was found to be +1′ 30″; the approximate latitude was 49°. Find the true latitude.

Number	Vertical Circle	Watch Time
1	24° 20′ 30″	11^h 10^m 14^s
2	24 20 30	11 12 30
3	24 21 00	11 14 30
4	24 21 30	11 16 50
5	24 20 50	11 18 50
6	24 20 30	11 20 54

Ans. 49° 4′ 14.4″

(3) Find the watch time of apparent noon on January 6, 1903, if the observer is in longitude $+2^h\ 3^m\ 18.1^s$ and his watch is $1^m\ 31.1^s$ slow on local mean time.

Ans. $12^h\ 4^m\ 13.8^s$

(4) At Washington, a watch is $9^m\ 11^s$ fast on standard eastern time; find the watch time of apparent noon on January 6, 1903, the longitude of Washington being $+5^h\ 8^m\ 15.75^s$ from Greenwich. Ans. $12^h\ 23^m\ 9.42^s$ P. M.

(5) Ten double altitudes of the sun were observed on the morning of January 7, 1903, a mercury horizon being used; one-half of the measurements were taken on the upper edge and one-half on the lower edge of the sun, and the watch time of each observation was recorded. The mean of the observed altitudes was 14° 10′ 58.2″; the mean of the recorded watch times was $9^h\ 9^m\ 13.2^s$ A. M. Find the true mean time and the watch error, the longitude being $-7^m\ 37^s$ and the latitude +39° 58′ 2″.

Ans. $\begin{cases} \text{Mean time, } 9^h\ 1^m\ 2.63^s \\ 8^m\ 10.6^s \text{ fast} \end{cases}$

(6) The following observations on the sun were made, as described in Art. 35, for the purpose of determining the azimuth of a line by the method of Art. 38. The latitude was +38° 53′ 18″, the declination of the sun was +13° 55′ 33″, the semi-diameter of the sun was 16′ 27″ and the correction due to index error of the vertical circle was −3′ 40″; find the azimuth.

Approximate Time P. M.	Vertical Circle	Horizontal Circle	Diagram of Field
$5^h\ 2^m$	29° 36′ 0″	25° 26′ 30″	
5 5	29 20 0	25 36 30	
5 6	29 2 0	25 46 0	
5 9	28 59 0	25 58 0	
5 10	28 45 0	26 2 0	
5 11	28 36 30	26 6 30	

Ans. 238° 45′ 50″

(7) The following observations for azimuth were made as described in Art. 40. The latitude being +40° 36′ 27″ and the declination of the sun +19° 54′ 5″, find the azimuth.

Telescope	Time A. M.	Vertical Circle	Horizontal Circle	Diagram of Field
Direct. . .	8^h 40^m	43° 09′ 00″	64° 48′ 00″	
Inverted. .	8 42	43 35 30	65 10 30	
Inverted. .	8 44	44 21 00	64 52 30	
Direct. . .	8 46	44 48 00	65 15 00	

Ans. 36° 44′ 18″

(8) Make a table of corrected declination settings for use in the field with a solar transit at San Francisco, between the hours of 3 and 4 P. M. on January 1, 1903, the longitude of San Francisco being +3^h 1^m 27^s from Washington, and the latitude being +36° 27′.

(9) The following observations were made on Polaris when this star was passing eastern elongation at Washington on January 3, 1903. The method was that described in Art. 50. The declination of Polaris being 88° 47′ 42″, and the latitude of Washington 38° 53′ 20″, find the azimuth of the line.

Telescope	Pointing on Star	Pointing on Mark
Direct	30° 8′ 30″	130° 9′ 40″
Inverted	117 36 10	217 37 30
Direct	224 10 50	324 12 10
Inverted	340 8 00	80 9 30

Ans. 101° 34′ 13″

(10) On the morning of July 1, a chronometer was compared with the observatory clock at a certain station. It was then carried to Washington and compared with the

observatory clock there, and on the morning of July 2 was brought back and compared with the first clock. It was known that the clock at the first station was 10^s fast, and was gaining 1.2^s each 24 hours. The clock at Washington was 5.8^s slow. Find the longitude of the station from Washington, the comparison being as follows:

	Observatory Clock	Chronometer
Station, July 1 . . .	$8^h\ 48^m\ 2^s$	$8^h\ 56^m\ 24^s$
Washington, July 1 .	18 13 36	21 14 30
Station, July 2 . . .	9 56 24	10 4 46

Ans. $2^h\ 52^m\ 15.6^s$

(11) The error of a watch on local mean time is $-2^m\ 4.8^s$ and on standard 75th-meridian time it is $+8^m\ 42.8^s$. What is the longitude west from Greenwich?

Ans. $4^h\ 49^m\ 12.4^s$

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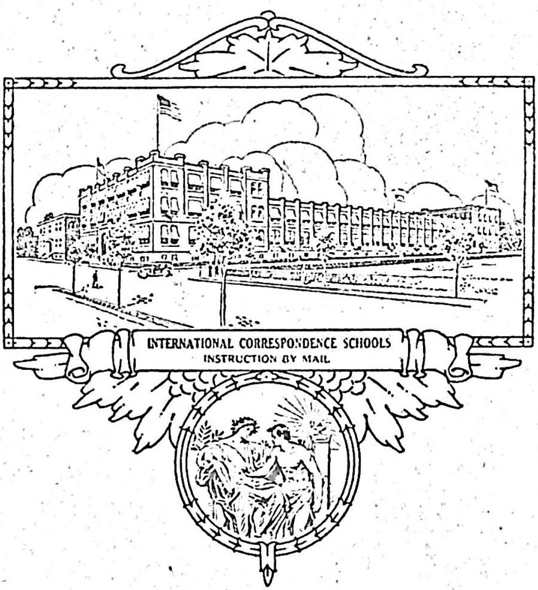
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